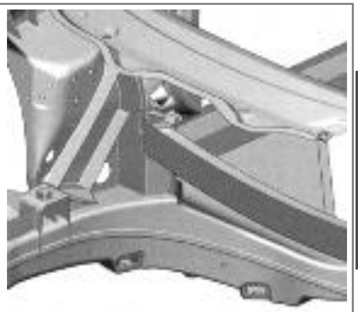

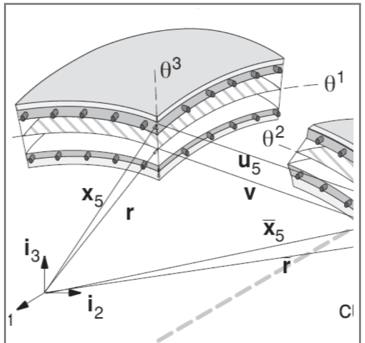

Developer Forum

Properties & Limits: Review of Shell Element Formulations

André Haufe[1], Karl Schweizerhof[1,2], Paul DuBois[3]

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Stuttgart

[2]Karlsruher Institute of Technology
Karlsruhe

[3]Independent Consultant
Offenbach

Motivation – from shells to solids?

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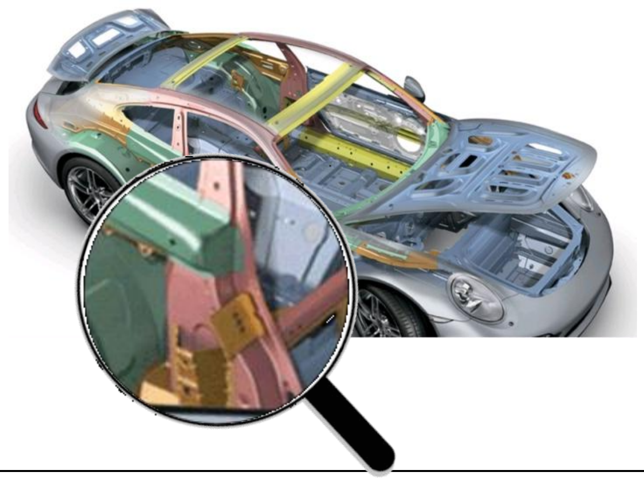
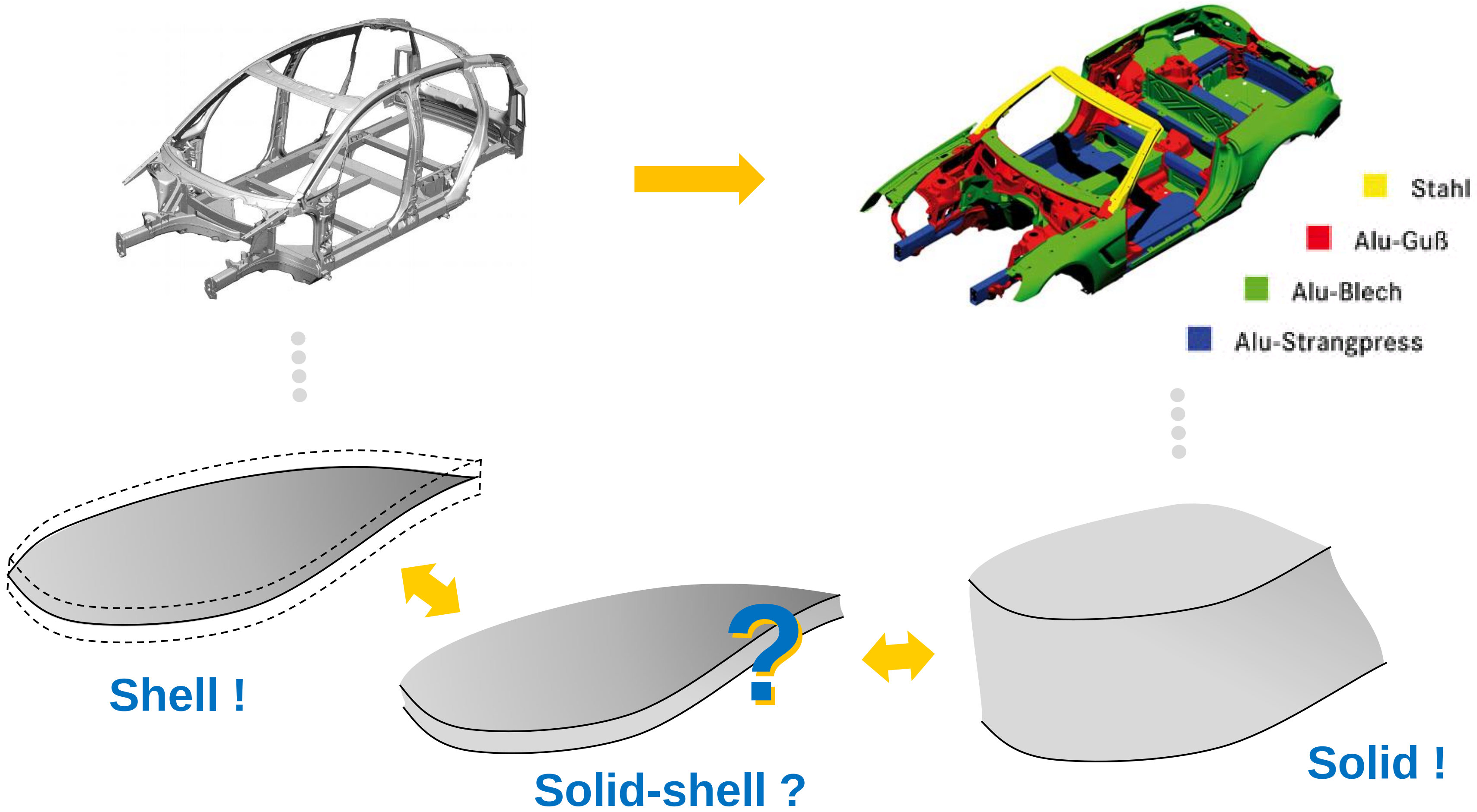
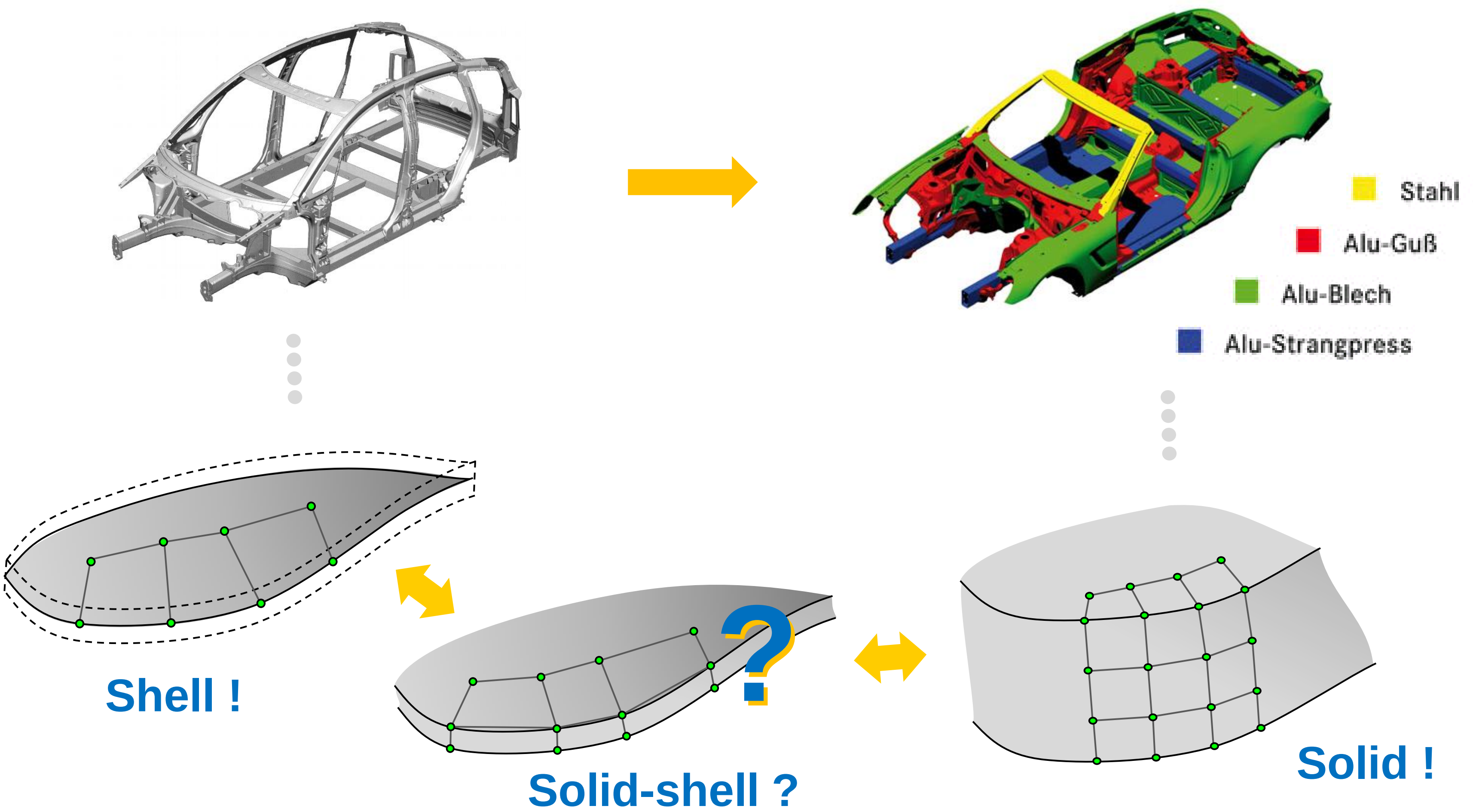


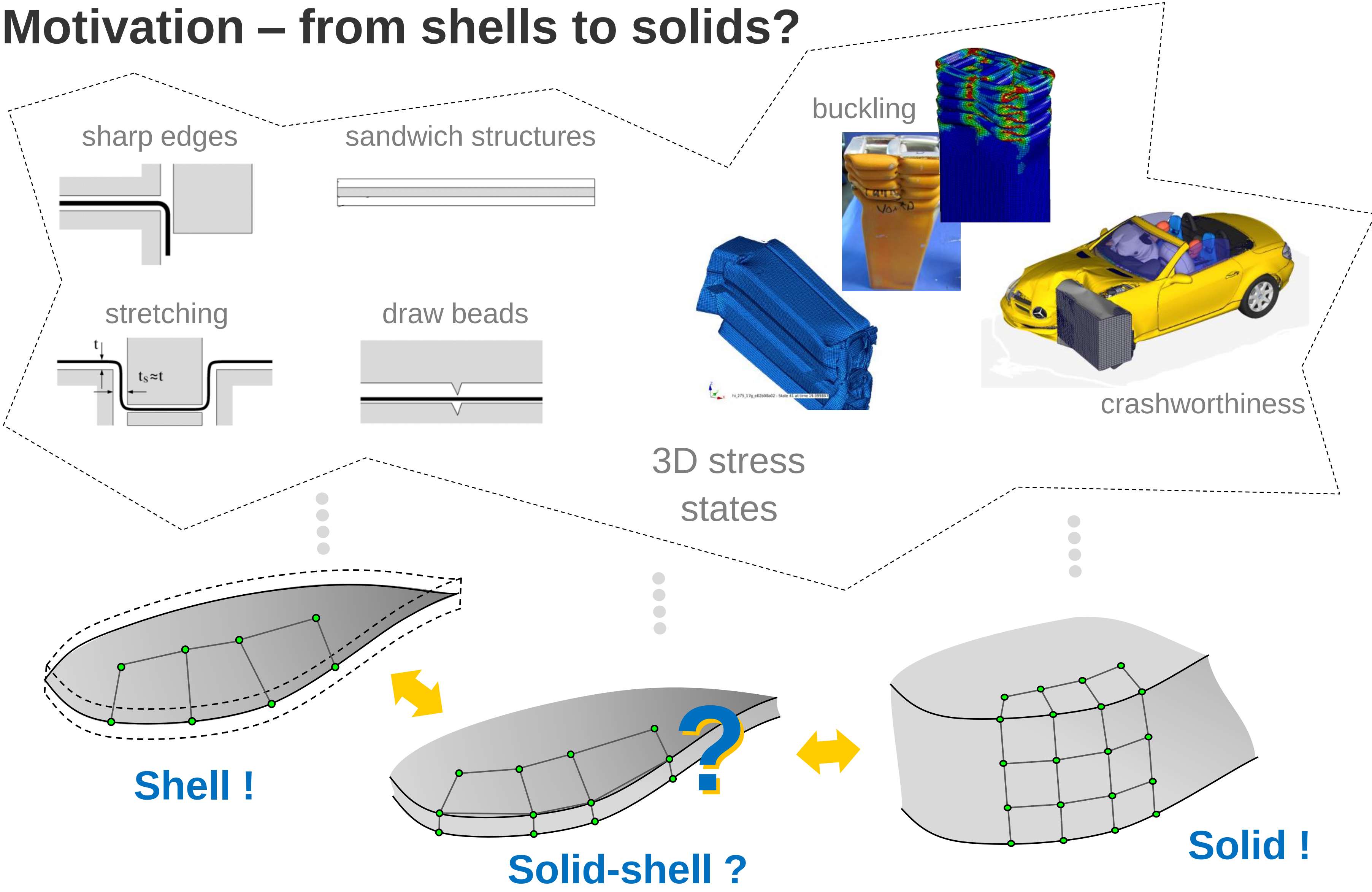
Motivation – from shells to solids?



Motivation – from shells to solids?



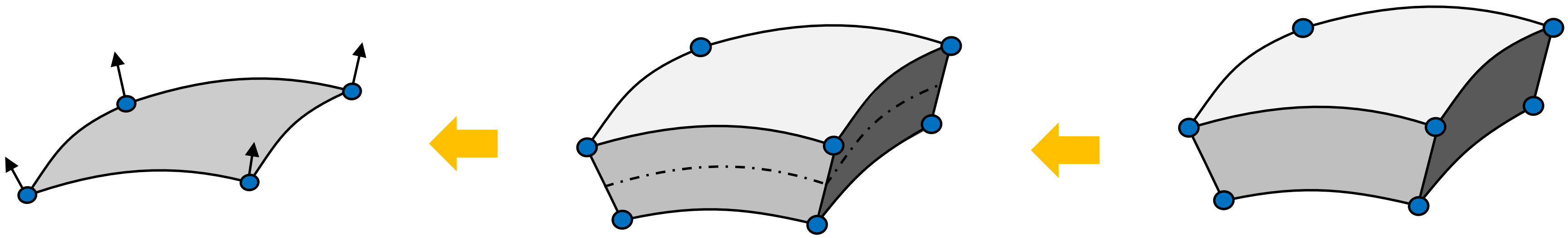
Motivation – from shells to solids?



Brief introduction

Derivation of shell formulation degenerated solid

[Ahmad, Irons and Zienkiewicz 1968]



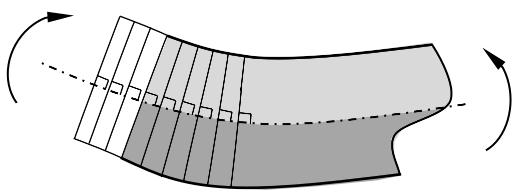
Shell assumptions (all quantities expressed w.r.t. mid surface)

Mid surface of characteristic plane

Solid element

Kirchhoff-Love [Bernoulli]

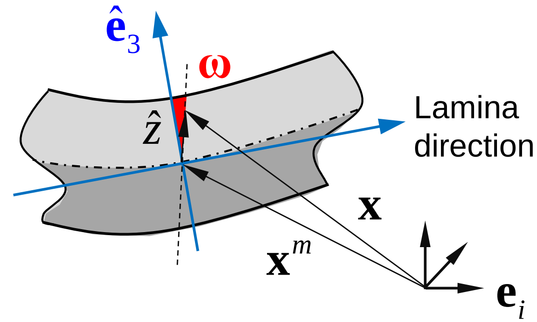
cross section straight, perpendicular and unstreched



Reisser-Mindlin [Timoshenko]

Mid-surface displacement plus rotations to describe plate (shell) deformation (shear deformation possible)

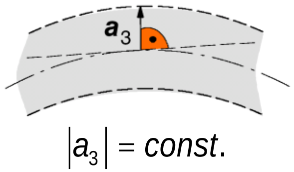
$$\mathbf{x} = \mathbf{x}^m + \hat{z}\,\hat{\mathbf{e}}_3 \times \boldsymbol{\omega}$$



Shell theories / Shell models

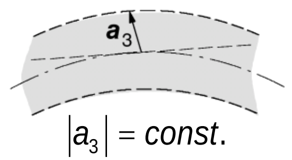
- **3-parameter shell model: Kirchhoff-Love**
  (cross section straight and unstreched,
  no shear deformations, i.e. normal to mid surface)

$$\sigma_{zz} = 0,\ \varepsilon_{zz} = 0$$
$$\gamma_{xz} = \gamma_{yz} = 0$$



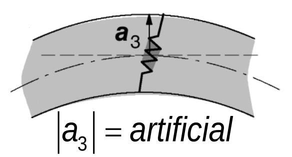
- **5-parameter shell model: Reissner-Mindlin**
  (cross section straight and unstreched,
  shear deformations possible)

$$\sigma_{zz} = 0,\ \varepsilon_{zz} = 0$$
$$\gamma_{xz} \neq 0;\ \gamma_{yz} \neq 0$$



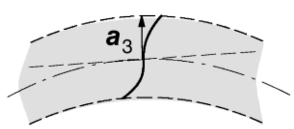
- **6- or 7-parameter shell model:**
  (cross section straight but stretchable)

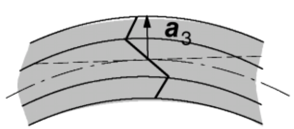
$$\sigma_{zz} \neq 0,\ \varepsilon_{zz} \neq 0$$
$$\gamma_{xz} \neq 0;\ \gamma_{yz} \neq 0$$



- **Higher order shell theory: multi-layer or -director:**
  (not straight and stretchable)

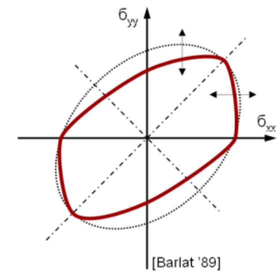
$$\sigma_{zz} \neq 0,\ \varepsilon_{zz} \neq 0$$
$$\gamma_{xz} \neq 0;\ \gamma_{yz} \neq 0$$

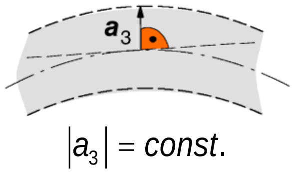


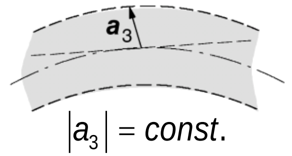


Shell theories / Shell models

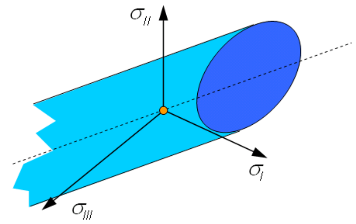
- **3-parameter shell model: Kirchhoff-Love**
  (cross section straight and unstreched,
  no shear deformations, i.e. normal to mid surface)
- **5-parameter shell model: Reissner-Mindlin**
  (cross section straight and unstreched,
  shear deformations possible)
- **6- or 7-parameter shell model:**
  (cross section straight but stretchable)
- **Higher order shell theory: multi-layer or -director:**
  (not straight and stretchable)

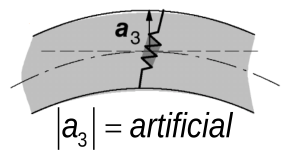


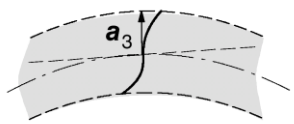


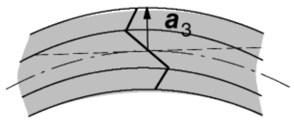


Reduced constitutive models
(2D *„plane stress“*)









Full 3D constitutive models

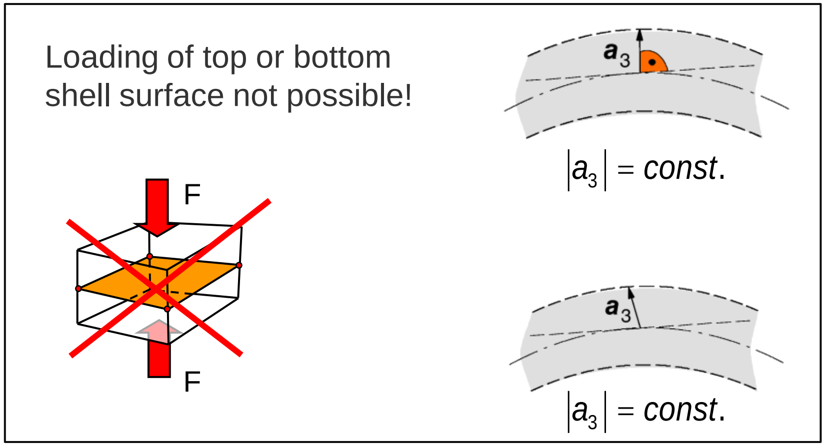
Shell theories / Shell models

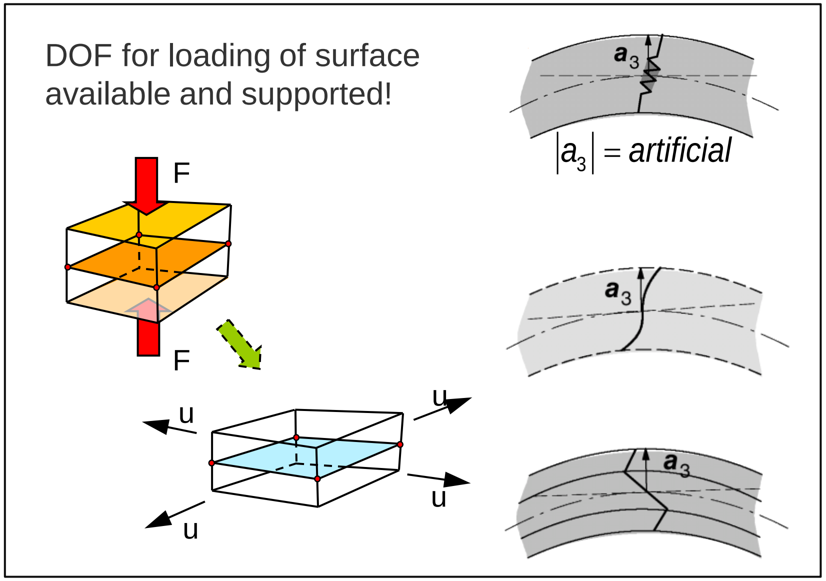
- **3-parameter shell model: Kirchhoff-Love**
  (cross section straight and unstreched,
  no shear deformations, i.e. normal to mid surface)

- **5-parameter shell model: Reissner-Mindlin**
  (cross section straight and unstreched,
  shear deformations possible)

- **6- or 7-parameter shell model:**
  (cross section straight but stretchable)

- **Higher order shell theory: multi-layer or -director:**
  (not straight and stretchable)





Welcome to the zoo!

What's in the zoo: SECTION_{T}SHELL

SECTION_SHELL

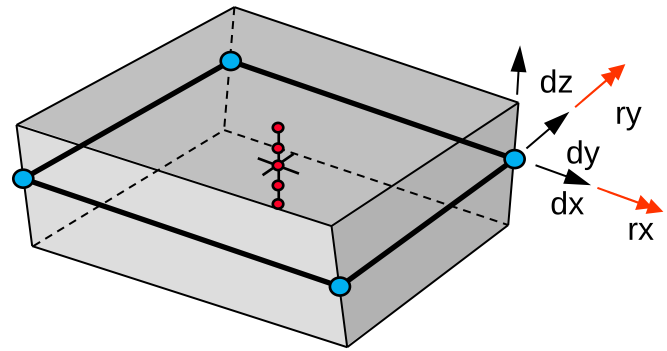
EQ.1: Hughes-Liu,
EQ.2: Belytschko-Tsay,
EQ.3: BCIZ triangular shell,
EQ.4: C0 triangular shell,
EQ.5: Belytschko-Tsay membrane,
EQ.6: S/R Hughes-Liu,
EQ.7: S/R co-rotational Hughes-Liu,
EQ.8: Belytschko-Leviathan shell,
EQ.9: Fully integrated Belytschko-Tsay membrane,
EQ.10: Belytschko-Wong-Chiang,
EQ.11: Fast (co-rotational) Hughes-Liu,
EQ.12: Plane stress (x-y plane),
EQ.13: Plane strain (x-y plane),
EQ.14: Axisymmetric solid - area weighted,
EQ.15: Axisymmetric solid - volume weighted,
EQ.16: Fully integrated shell element
EQ.17: Fully integrated DKT, triangular shell element,
EQ.18: Fully integrated linear DK quadrilateral/triangular shell
EQ.20: Fully integrated linear assumed strain C0 shell
EQ.21: Fully integrated linear assumed strain C0 shell (5 DOF).
EQ.22: Linear shear panel element (3 DOF per node)
EQ.23: 8-node quadratic quadrilateral shell
EQ.24: 6-node quadratic triangular shell
EQ.25: Belytschko-Tsay shell with thickness stretch.
EQ.26: Fully integrated shell with thickness stretch.
EQ.27: C0 triangular shell with thickness stretch.

SECTION_TSHELL

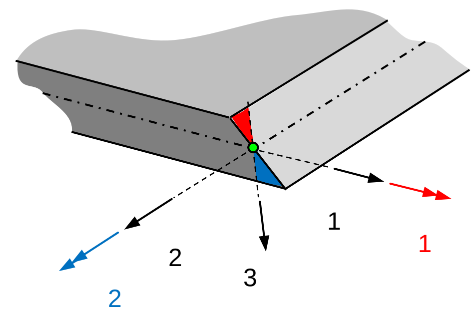
EQ.1: one point reduced integration (default),
EQ.2: selective reduced 2 x 2 in plane integration.
EQ.3: assumed strain 2 x 2 in plane integration
EQ.5: assumed strain reduced integration

Hughes-Liu shell formulation

ELFORM=1 and 11



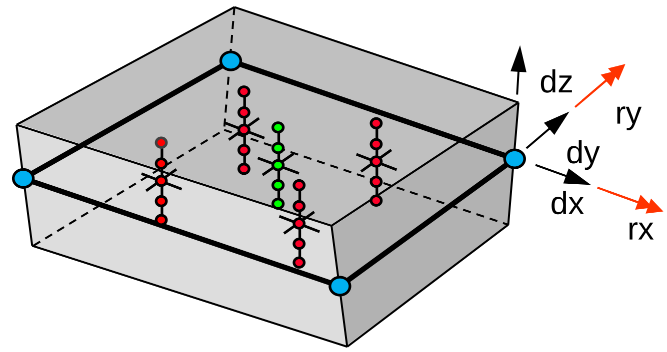
- Based on degenerated continuum element formulation (5 DOF in local coordinate system yield globally 6 DOF)
- Computationally costly (compared to ELTYP=2) but effective when large deformations are expected
- Warped configurations are treated correctly.
- Bi-linear nodal interpolation
- One-point integration for efficiency reasons
- Hourglass control to counterbalance zero energy modes
- Does not pass the patch test



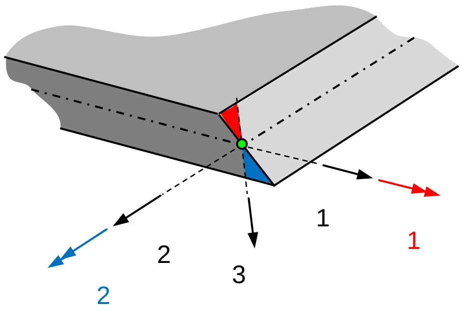
- The fast (co-rotational) Hughes-Liu formulation (ELFORM=11) is the same as the Hughes-Liu ELFORM=1 except this formulation uses the co-rotational system.

Hughes-Liu shell formulation

ELFORM=6 and 7



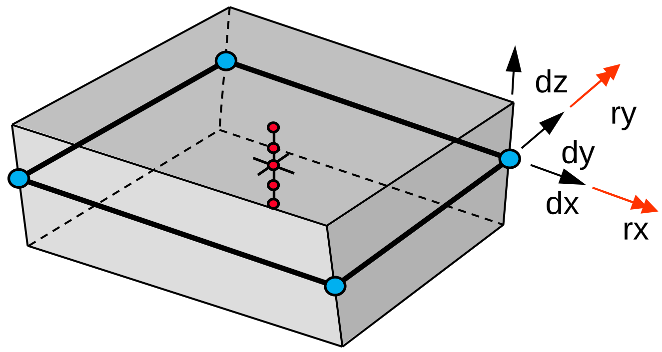
- Based on degenerated continuum element formulation (5 DOF in local coordinate system yield globally 6 DOF)
- Bi-linear nodal interpolation
- Selective reduced integration (SRI) is used to avoid most hourglass modes.
- Computationally even more costly (3-4) than ELFORM=1
- Bending hourglass modes are still possible



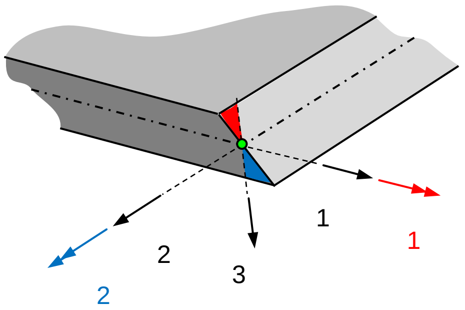
- The selectively reduced co-rotational Hughes-Liu element formulation (ELFORM=7) is the same as the selectively Hughes-Liu except it uses the co-rotational system.

Belytschko-Lin-Tsay shell formulation

ELFORM=2 (default element)



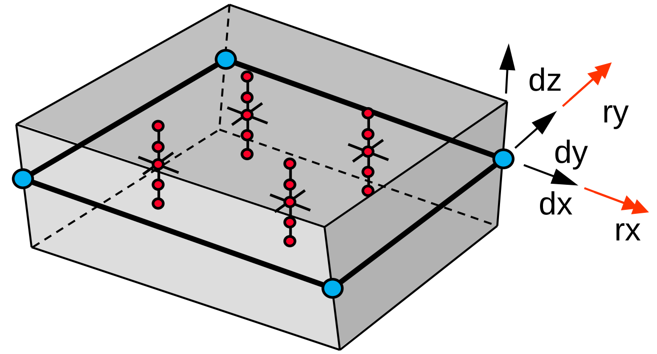
- Based on Reissner-Mindlin kinematic assumption
  (5 DOF in local coordinate system yield globally 6 DOF)
- Extremely effective: Formulated in velocity strains (rate of deformations) and Cauchy stresses (…)
- Bi-linear nodal interpolation
- One-point integration for efficiency reasons
- Updated, co-rotational Lagrangean formulation
- Hourglass control to counterbalance zero energy modes
- warping stiffness (BWC flag on *CONTROL_SHELL)
- Warpage may be an issue; do not use in coarse models
- Nodal fiber vectors for improved warping stiffness
- Fastest kid in town but does not pass the patch test



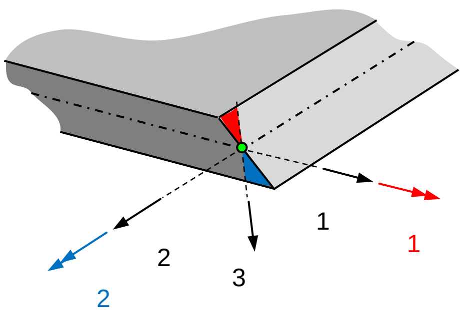
- The Belytschko-Wong-Chiang formulation (ELFORM=10) is the same as the Belytschko-Tsay except the shortcomings in warped configuration area avoided. Costs about 10% more.
- The Belytschko-Leviathan shell formulation (ELFORM=8) is similar to the Belytschko-Wong-Chiang with one-point quadrature but it uses physical hourglass control, thus no hourglass control parameters need to be set by the user.

Fully-integrated shell formulation

ELFORM=16

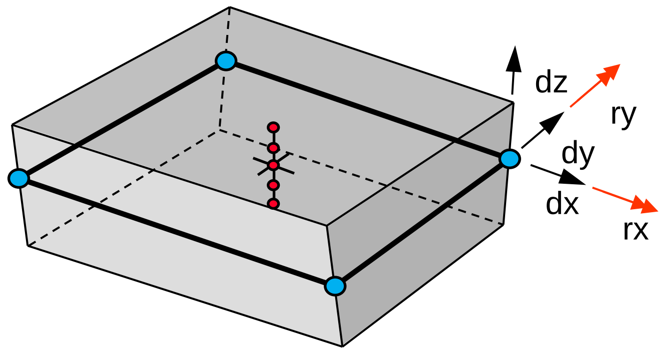


- Based on Reissner-Mindlin kinematic assumption
  (5 DOF in local coordinate system yield globally 6 DOF)
- 2x2 integration in the shell plane
- Bathe-Dvorkin (AS) transverse shear correction
  eliminates W-mode hourglassing
- Hourglass type 8 adds warping stiffness (may improve convergence)
- Least expensive of 2x2 integrated elements
- 2-3 times more expensive than the Belytschko-Tsay shell
- Does not degenerate to triangle
- Objective stress update used
- Recommended for implicit simulations.

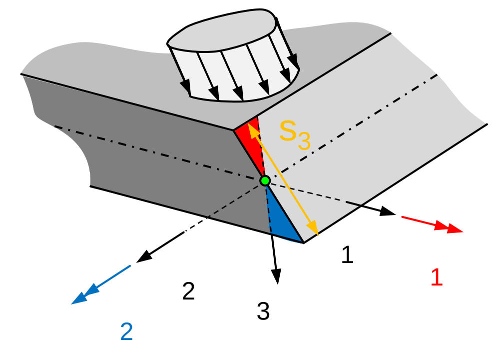


Thickness enhanced shell formulation

ELFORM=25 (the thick-thin shells)



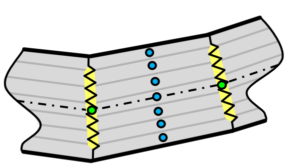
- Based on Reissner-Mindlin kinematic assumption (5 DOF in local coordinate system yield globally 6 DOF)
- Additional feature of linear strain through the thickness. The latter is important to avoid "Poisson locking" in bending modes of deformation.
- Thickness stretch requires 3D constitutive model
- Bi-linear nodal interpolation
- One-point integration for efficiency reasons
- Updated, co-rotational Lagrangean formulation
- Hourglass control to counterbalance zero energy modes



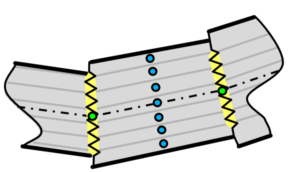
- Option to decouple thickness field across shell edges:

SECTION_SHELL, IDOF:

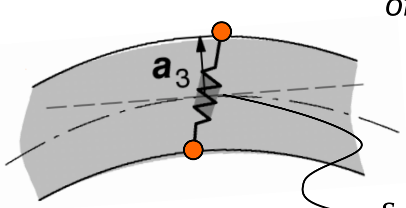
EQ.1: Thickness field is continuous (recommended for sheet metal forming)



EQ.2: Thickness field is discontinuous (default)



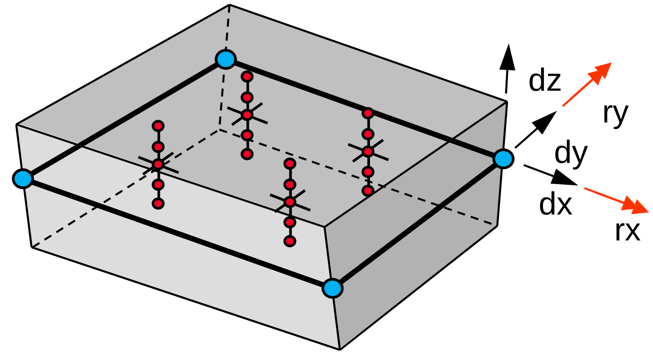
Loading and contact of surface possible



$$s_3 = \frac{\xi_3}{2} t_3 + (1 - \xi_3^2) q_3$$

Thickness enhanced shell formulation

ELFORM=26 (the thick-thin shells)

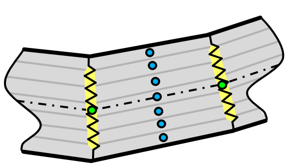


- Based on Reissner-Mindlin kinematic assumption
  (5 DOF in local coordinate system yield globally 6 DOF)
- 2x2 integration in the shell plane
- Bathe-Dvorkin (AS) transverse shear correction eliminates W-mode hourglassing
- Additional feature of linear strain through the thickness. The latter is important to avoid "Poisson locking" in bending modes of deformation.
- Thickness stretch requires 3D constitutive model
- Hourglass type 8 adds warping stiffness

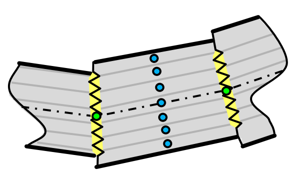
- Option to decouple thickness field across shell edges:

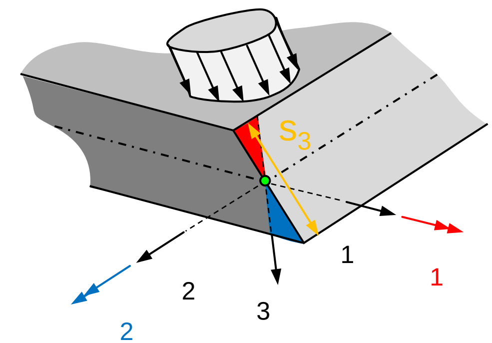
SECTION_SHELL, IDOF:

EQ.1: Thickness field is continuous (recommended for sheet metal forming)

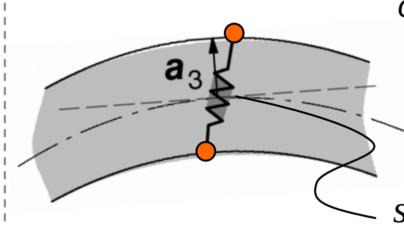


EQ.2: Thickness field is discontinuous (default)





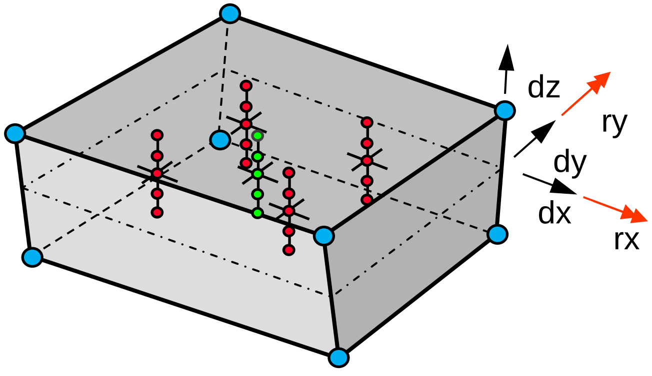
Loading and contact of surface possible



$$s_3 = \frac{\xi_3}{2} t_3 + (1-\xi_3^2) q_3$$

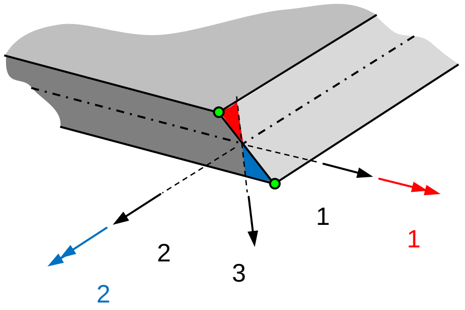
TSHELL elements in LS-DYNA

ELFORM=1 and 2 (the thin-thick shells)



- Despite the solid discretization, the classical Reissner-Mindlin kinematic is employed!
- Hence the constitutive law is **plane stress** based and the thickness change is based on the Poisson-number.
- **ELFORM=1** uses one point quadrature in the shell plane.
- **ELFORM=2** uses SRI in the shell plane.

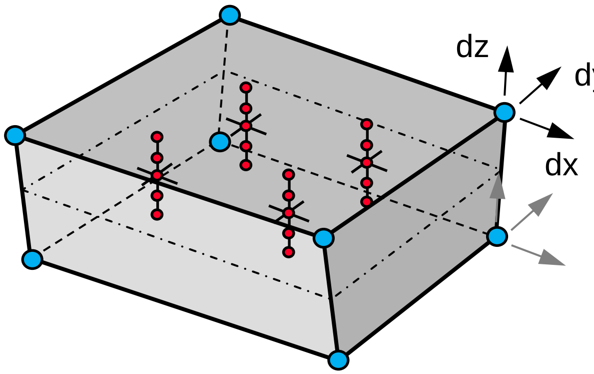
- Both formulations may capture bending as well as classical shell elements (in fact they are classical shell elements…).
- **NIP** defines the number of integration point in thickness direction. It will be modified internally for ELFORM=1 to an uneven number.
- Shear correction factor SHRF is available (and useful).
- Element stacking is not recommended!!



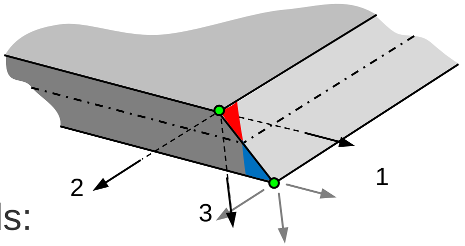
Nodal rotations may be constructed via a automatically generated mid-surface and relative displacements of upper and lower surface nodes

TSHELL elements in LS-DYNA

ELFORM=3 and 5 (the thick-thick shells)



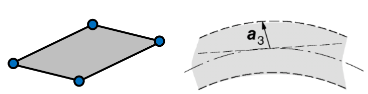
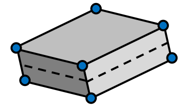
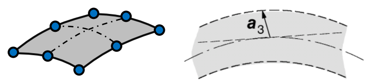
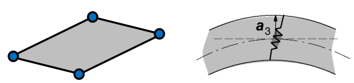
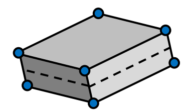
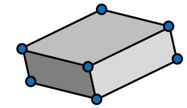
- ELFORM=3/5 are shell-like solid elements, i.e. solid formulations with a strong emphasis on shell properties.
- Hence the constitutive law is fully 3D based and the thickness change come naturally from corresponding degrees of freedom.
- Both use co-rotational formulations.
- **ELFORM=3** uses assumed strain 2x2 quadrature in the shell plane.
- **ELFORM=5** uses assumed strain and SRI in the shell plane.



- Both formulations may capture bending only in a stacked discretization properly. A minimum of 2 shells is recommended.
- Hence SHRF is not used (since not meaningful) except for special treatment in certain sheet metal forming constitutive models: MAT_33/36/133/135/243
  - ELFORM=3 applied constant shear stiffness across thickness
  - ELFORM=5 may be switched from constant to parabolic with TSHEAR.

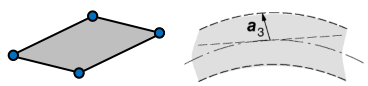
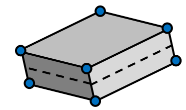
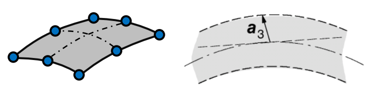
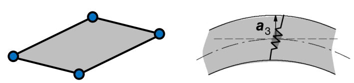
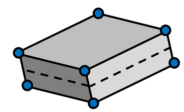
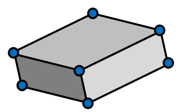
- ELFORM=5 uses an assumed strain method to capture the complex Poisson's effects and through thickness stress distribution in layered composites (see composite seminar and LAMSHT)

Overview on element formulations

	*SECTION_SHELL	*SECTION_TSHELL	*SECTION_SOLID 8 noded
Thin shell S110	 2, 16 1, 6, 7, 10, 11	 1, 2	-
Thin shell S220	 23	-	-
Thick shell[1)] **or solid** S111	 25, 26	 3, 5	 1, 2, -1, -2
Enn0 (generalized shell)	1000	-	-
Eii0 (Isogeometric shell)	201	-	-

1) A shell formulation is denoted „thin shell“ if the neutral axis (plane) stays in the middle of the section. Contrary to this it is denoted „thick shell“ if the neutral axis is allowed to move during the deformation process, i.e. in bending. Typically thick shells of this type require 3D constitutive models.

Overview on element formulations

	*SECTION_SHELL	*SECTION_TSHELL	*SECTION_SOLID 8 noded
Thin shell S110	 2, 16 1, 6, 7, 10, 11	 1, 2	-
Thin shell S220	 23	-	-
Thick shell[1)] **or solid** S111	 25, 26	 3, 5	 1, 2, -1, -2

The critical time steps for shells are computed as follows, where c_s is the corresponding speed of sound:

thin-thin-shells

$$c_s = \sqrt{\frac{E}{(1-\nu^2)\rho}}$$

$$\Delta t_{e,crit} = \frac{A}{c_s\, l_{\max}} \quad or \quad \frac{A}{c_s\, d_{\max}}$$

thick-thin-shells

$$c_s = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}}$$

$$\Delta t_{e,crit} = \frac{A}{c_s\, l_{\max}} \quad or \quad \frac{A}{c_s\, d_{\max}}$$

thin-thick-shells

$$c_s = \sqrt{\frac{E}{(1-\nu^2)\rho}}$$

$$\Delta t_{e,crit} = \frac{V_{ele}}{c_s\, A_{ele,\max}}$$

thick-thick-shells *

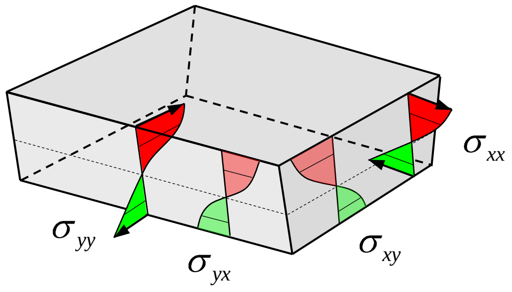
$$c_s = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}}$$

$$\Delta t_{e,crit} = \frac{V_{ele}}{c_b\, A_{ele,\max}}$$

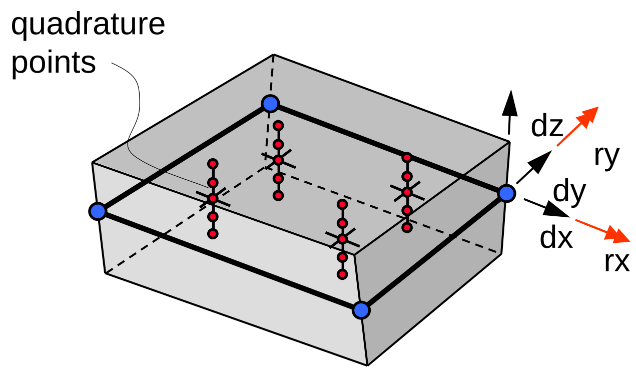
**Also true for solids; bulk viscosity may influence time step too.*

Additional things to remember

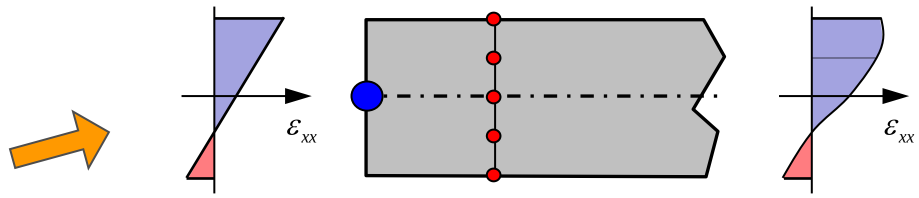
Integration in thickness direction (classical shells)



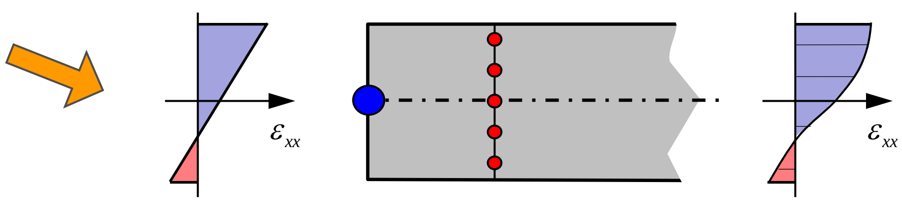
Reisser-Mindlin-kinematic assumed



fully integrated shell element



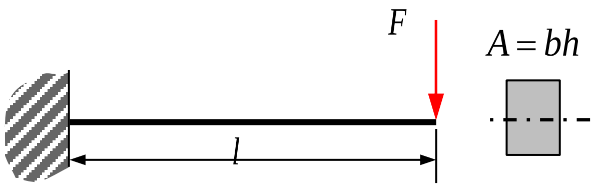
Lobatto-integration
(quadrature on surface)

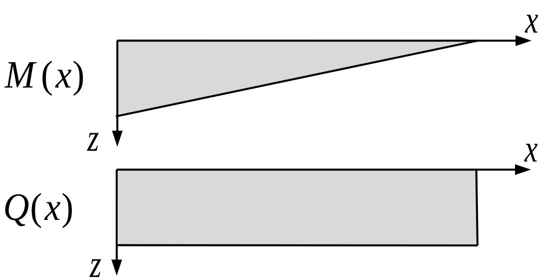


Gauss-integration
(extrapolation to surface necessary)

Classical shell elements: Shear distribution and LAMSHT

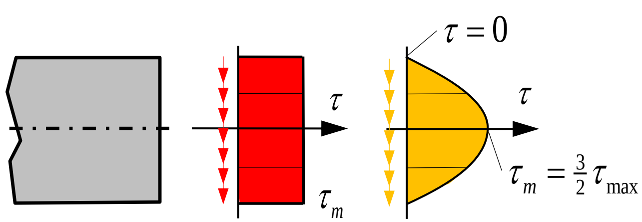
Shear correction factor (elastic, isotropic, uniform sections)





$$M(x) = \int_A \sigma(x)\, z\, dA$$

$$Q(x) = \int_a \tau(x)\, dA$$



$$\tau_m(x) = \frac{Q(x)}{A(x)} \qquad \tau(x,z) = \frac{3Q(x)}{2A}\left(1 - \frac{4z^2}{h^2}\right)$$

Correction of shear area possible: $A_{shear} = \kappa A_{real} = \frac{5}{6} A_{real}$

If the section is neither elastic, nor uniform the shear correction factor is not correct.

This is especially the case for sandwich structures where large stiffness jumps across the thickness (dissimilar materials) are part of the design. In this case laminated shell theory is mandatory:

- LAMSHT corrects for the incorrect assumption of uniform constant shear strain through the thickness of the shell.
- Without LAMSHT a sandwich composite will generally be too stiff.
- LAMSHT=1 in *control_shell invokes laminated shell theory.
- Mat_layered_linear_plasticity (114) is a plasticity model much like mat_024 but which includes LST.

Shell element control recommendations

CTRL_SHELL

Shell thickness change option for deformable shells (ISTUPD).

EQ.0: no thickness change

EQ.1: membrane straining causes thickness change in classical shell elements. This option is very important in sheet metal forming or whenever membrane stretching is important.

EQ.2: membrane straining causes thickness change in 8 node thick shell elements, types 1 and 2.
The types 3 and 5 thick shells are a continuum based shells and thickness changes are always considered.

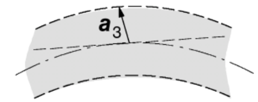
EQ.3: options 1 and 2 apply.

EQ.4: option 1 applies, but the elastic strains are neglected for the thickness update. This option only applies to the most common elastic-plastic models. For crash analysis, neglecting the elastic component of the strains may improve energy conservation and stability.

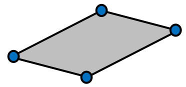
Warping stiffness for Belytschko-Tsay shells (BWC):

EQ.1: Belytschko-Wong-Chiang warping stiffness added.

EQ.2: Belytschko-Tsay (default).

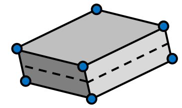


Classical shell has no DOF in thickness direction!

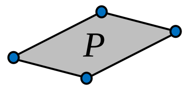


Update via Poisson behavior:

$$\Delta\varepsilon_{33} = \frac{\nu}{\nu - 1}\ \Delta\varepsilon_{11} + \Delta\varepsilon_{22}$$



Thick-thick-shell has corresponding DOF



Update via Poisson behavior

$$\Delta\varepsilon_{33} = \frac{\nu}{\nu - 1}\ \Delta\varepsilon_{11}^{p} + \Delta\varepsilon_{22}^{p}$$

where often $\nu = 0.5$

Shell element accuracy recommendations

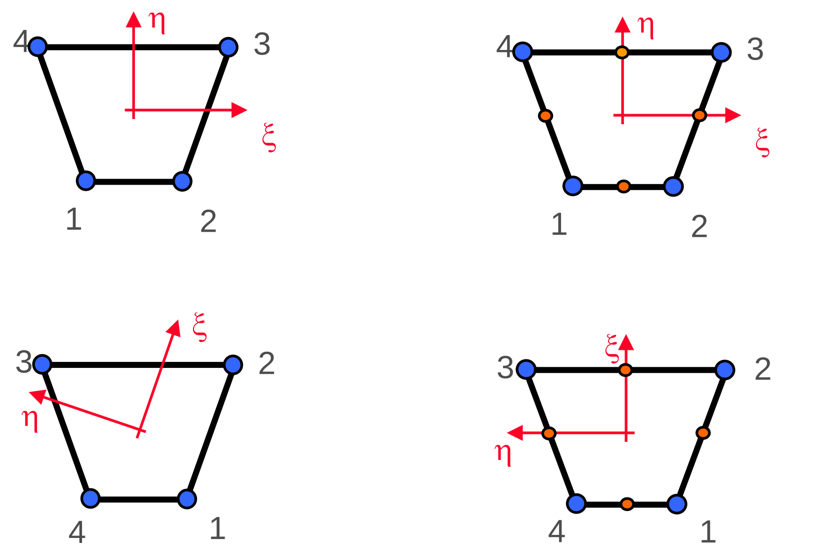
CTRL_ACCURACY

Objective stress update (OSU):

- include second order terms in stress state
- required for large rotational motion and/or large time steps due to mass-scaling
- requires mid-step geometry evaluation (n+1/2)
- higher accuracy but more expensive

Invariant node numbering option (INN):

- By default the local x-direction is given by the element edge from 1st to 2nd node of element
- Permuting nodes with default numbering creates different local system in non-rectangular elements
- permuting nodes with invariant option shifts local system in 90 degree increments, regardless of element shape

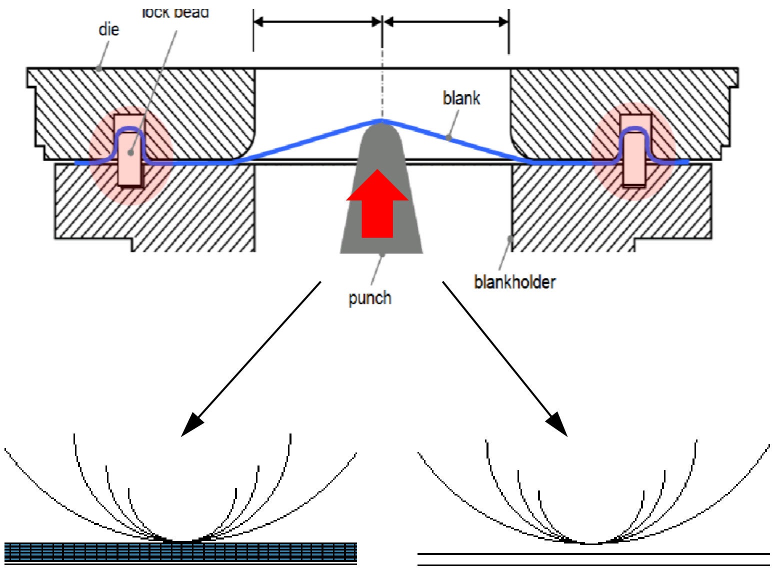


default numbering (`INN = 1`)	*invariant numbering* (`INN = 2`)

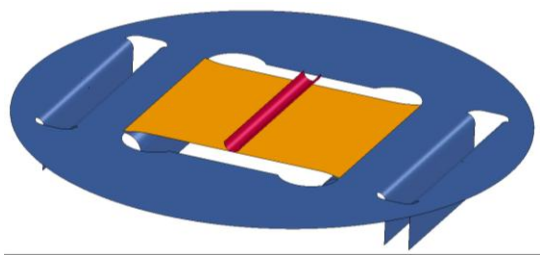
Example 1

Stretch bending test with various materials and discretization

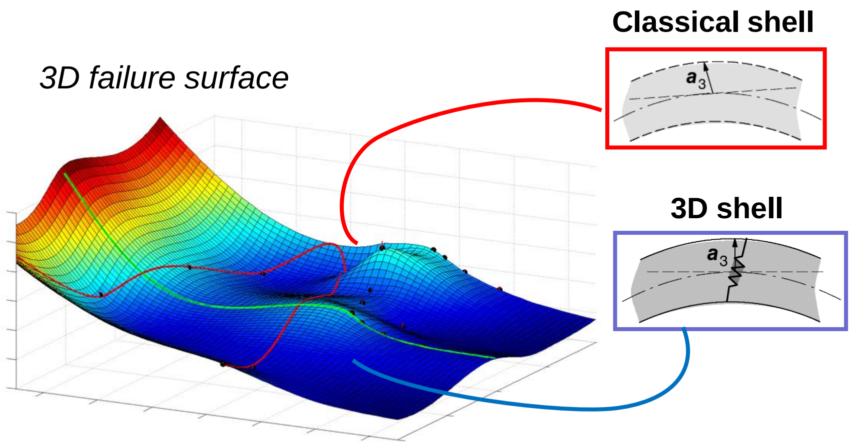
Funding by RFCS greatly acknowledged



Element Type	Shells	Solids
Element formulation	2/ 25	-1
Number of integration points over thickness	6	1
Number of elements across thickness direction	1	6
Element edge length	0,25mm	0,25mm
Selective mass scaling	✓	✓
Number of integration points that should fail before element fails	5	1

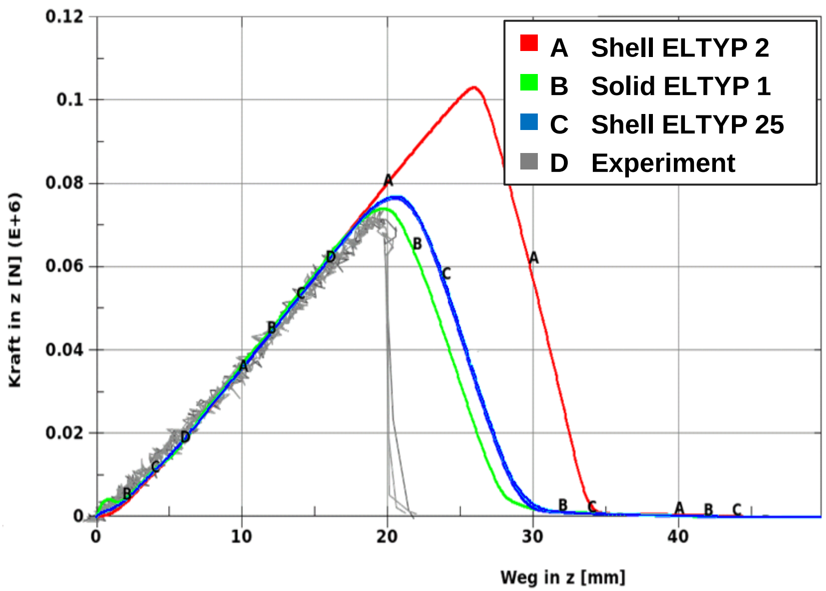


Different radii r05/r07/r10/r20
in shells and solids

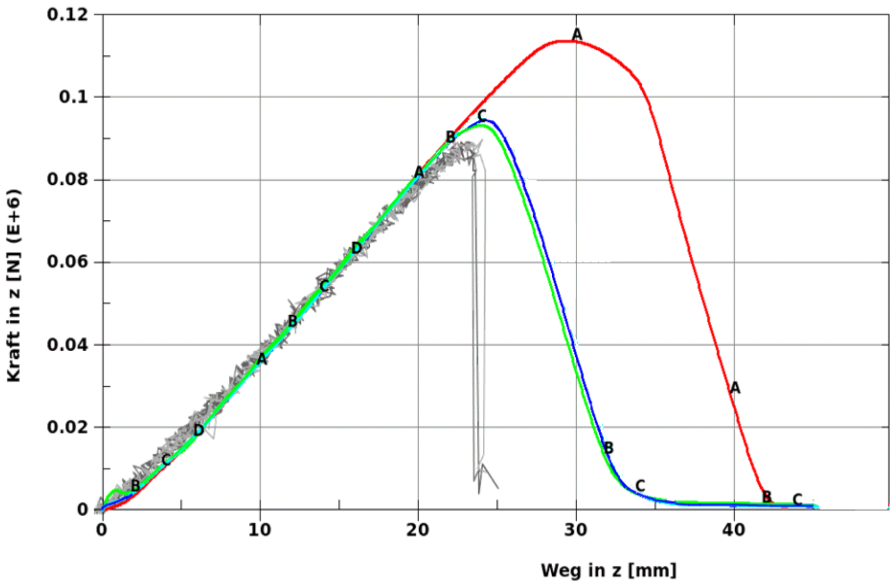


Stretch bending: CP800 and R05 / R10

CP800 and R05



CP800 and R10



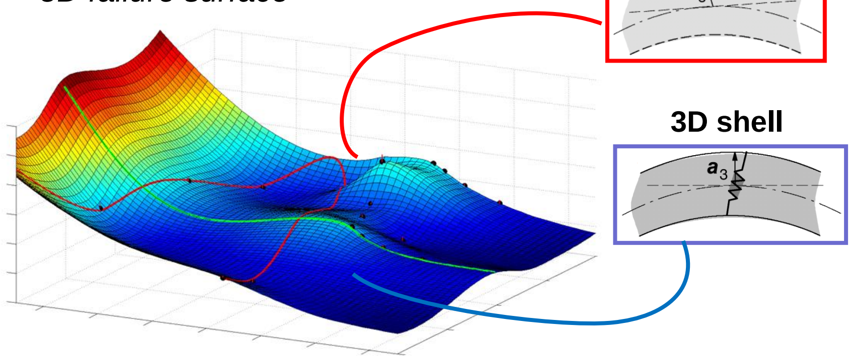
Failure data has been calibrated for plane stress states using DIEM, TYP=1 shear failure model:

$$\varepsilon_D^p = \varepsilon_D^p(\theta, \dot{\varepsilon}^p) \quad \text{where} \quad \theta = (q + k_S\, p)/\tau$$

$$\text{and} \qquad \tau = \sigma_{\text{major}} - \sigma_{\text{minor}} \; /2$$

The data was then converted to GISSMO-curves and applied directly without any other modification to the application shown.

3D failure surface



➩ ***Recent investigations show, that ELTYP=16 with IDOF=3 delivers results of similar accuracy as B and C!!***

Example 2

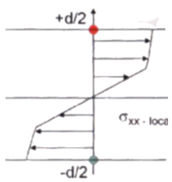
Limits of shell elements in bending

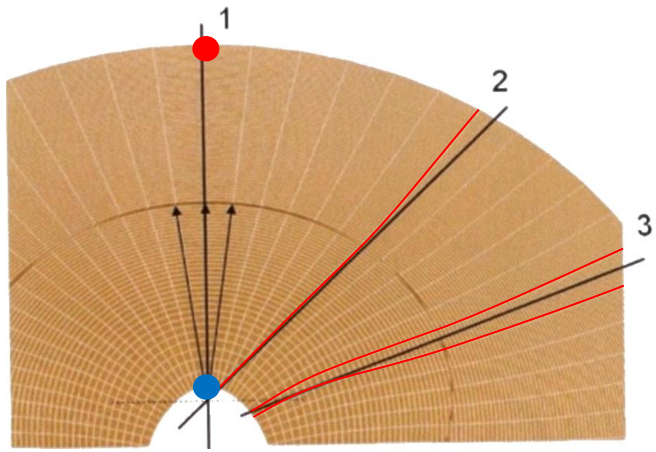
[Dissertation Michael Fleischer, TUM, Germany]

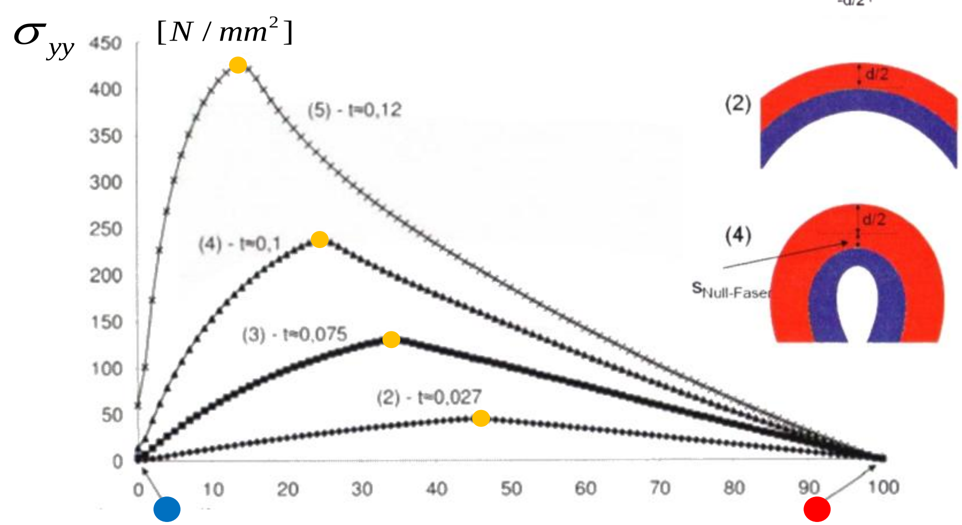
Virtual ring-tension test

$$R_{M,RZ} = 5mm,\ d = 1.0mm,\ l_c = 1mm$$

Fine discretization with solid elements:
Possible violation of Bernoulli hypothesis (straight sections remain straight)







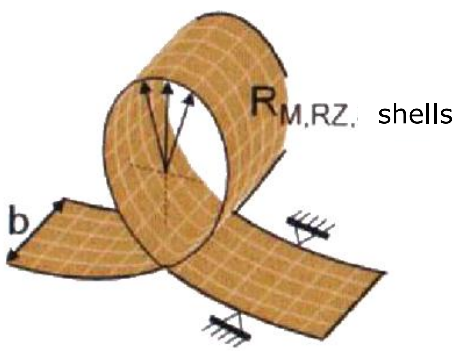
Limits of shell elements in bending

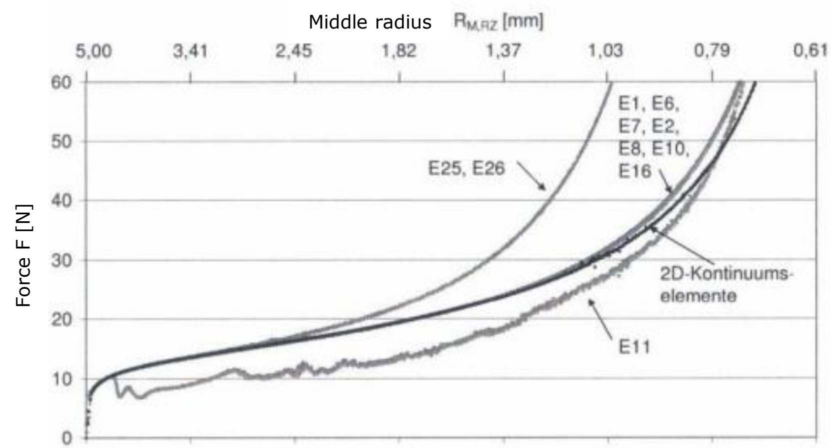
[Dissertation Michael Fleischer]

Virtual ring-tension test

$$R_{M,RZ} = 5mm,\ d = 1.0mm,\ l_c = 1mm$$

Discretization with different shell formulations:
Possible violation of Bernoulli hypothesis (straight sections remain straight)





Recommendation:
Geometrical limit (justified by force & deformation)

$$\frac{R_{M,RZ}}{d} \geq 1$$

FIN