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Element-Free-Galerkin Method (EFG) in LS-DYNA — Implementation and Applications

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The 3rd German LS-DYNA Forum

Outline

LS-DYNA

1. **Overview of LS-DYNA-EFG**
   - Implementation of LS-DYNA-EFG
   - Industrial Applications
2. **Recent Developments of LS-DYNA-EFG**
   - Inclusion of Different EFG Background Elements
   - Parallel EFG Computation
   - Implicit EFG Shells
3. **Conclusion**

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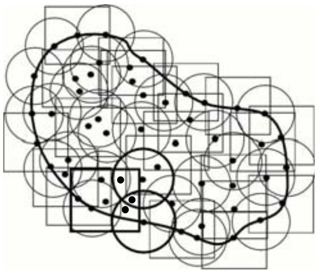
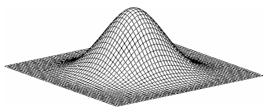
1. Overview on LS-DYNA-EFG

What is the Meshfree/Meshless/Particle Method ?

LS-DYNA

— No mesh is needed and shape functions are constructed from sets of particles





Meshfree Shape Function

- **Meshfree Method**
  - Meshfree Collocation Method
    - **Smooth Particle Hydrodynamics (SPH) [Monaghan 1977]**
    - Finite Point Method [Onate et al.1996]
  - Meshfree Galerkin Method
    - **Element Free Galerkin (EFG) [Belytschko et al. 1994]**
    - **Reproducing Kernel Particle Method (RKPM) [Liu et al. 1995]**
    - Partition of Unity Method [Babuska and Melenk 1995]
    - HP-Clouds [Duarte and Oden 1996]
    - Free-Mesh Method [Yagawa et al. 1996]
    - Natural Element Method [Sukumar et al.1998]
    - Meshless Local Petrov-Galerkin Meshfree Method(MLPG) [Atluri et al.1998]
    - Local Boundary Integral Equation (LBIE) [Atluri et al. 1998]
    - Finite Sphere Method [1998] …
- **(FEM, Control Volume, BEM …) + Meshfree Method**
  - **Coupled FEM/Meshfree Method [1995]**
  - Extended FEM Method [1999]
  - Finite Particle Method [1999]

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EFG Approximation

LS-DYNA

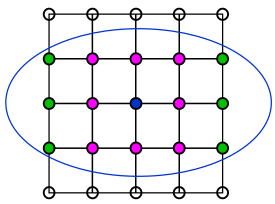
$$u^h(x) = \sum_{I=1}^{NP} \overline{w}_a^{[n]}(x - x_I)\, u(x_I)\, \Delta x_I$$

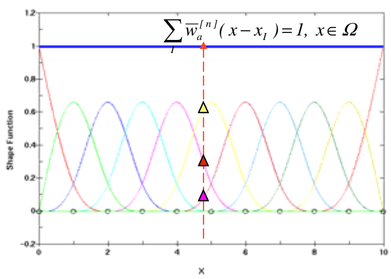
Moving Least-Squares approximation or Reproducing Kernel approximation

$$\overline{w}_a^{[n]}(x - x_I) = \underbrace{\boldsymbol{H}^{[n]^T}(0)\boldsymbol{M}^{[n]^{-1}}(x)\boldsymbol{H}^{[n]}(x - x_I)}_{n\text{-th order completeness}}\ \underbrace{w_a(x - x_I)}_{\text{weighting function}}$$

$$\overline{w}_{aI}^{[n]}(x_J) \neq \delta_{IJ}$$

$$\boldsymbol{A}^{-T}\boldsymbol{M}\boldsymbol{A}^{-1}\Delta\ddot{\boldsymbol{d}} + \boldsymbol{A}^{-T}\boldsymbol{K}\boldsymbol{A}^{-1}\Delta\boldsymbol{d} = -\boldsymbol{A}^{-T}\boldsymbol{R}$$





- Higher-order approximation
- More neighboring nodes
- Special treatment on B.C.
- Complicated domain integration (rely on background elements)

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Mixed Formulation for Nearly Incompressible Material LS-DYNA

Hellinger-Reissner variational principle

$$\Pi(u,p) = \int_\Omega \frac{1}{2} u_{(i,j)} \sigma_{ij} d\Omega - \int_\Omega \frac{1}{2} p u_{i,i} d\Omega - W^{ext}$$

$$u_{i,i} + p/\lambda = 0$$

Discrete equations

$$\begin{bmatrix} \overline{K} & G \\ G^T & M \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} \overline{F} \\ 0 \end{bmatrix}$$

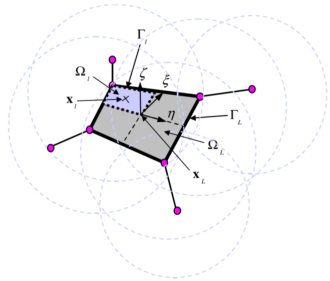
$$\left(\overline{K} - GM^{-1}G^T\right)u = \overline{F}$$

$$p^{(P)} = -M^{-1}G^T u$$

$$G_{IJ} = \int_\Omega \tilde{B}_I^{vT} \Psi_{,J} d\Omega$$

$$\tilde{\varepsilon}^d = \sum_I \tilde{B}_I^d \hat{d}_I$$

$$\overline{F}_I^{int} = \int_\Omega \tilde{B}_I^T \left(\sigma + p^{(P)}\right) d\Omega$$



Smoothed Gradient Matrix

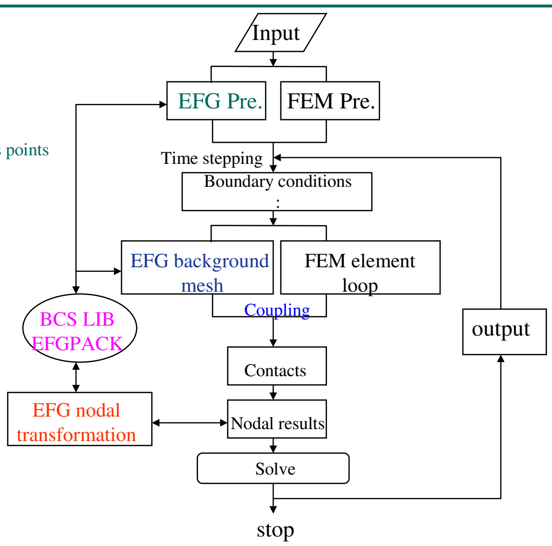
$$\tilde{B}_I(x_I) = \frac{1}{V_I} \int_{\Omega_I} B_I d\Omega$$

Overall Flowchart LS-DYNA

- surface projection for shell
- lump mass
- nodal transformation matrix
- neighboring information
- derivative shape functions at stress points

- mixed formulation if required
- assumed deformation gradient
- constitutive equations
- internal forces
- stiffness matrix

- generalized velocity
- generalized displacement



LS-DYNA-EFG for Industrial Applications

LS-DYNA

Mesh-free Basic Features

1. Smoother stress and strain
2. Less sensitive to the discretization
3. No hourglass control
4. Higher accuracy
5. Easier adaptivity
6. Higher CPU
7. More memory
8. More difficult in theory
9. More developments and refinements on theory

- **Solid** (LS-DYNA)
  EFG Plane strain #43
  EFG Axisymmetric #44
  EFG 3D solid #41
  - Rubber industry
  - Highly compressible foam
  - Human safety
  - Forging simulation
- **Shell** (LS-DYNA)
  EFG shell #41
  EFG shell #42
  - Metal Forming
  - Crashworthiness
- **Fluid**
  - Compressible/Incompressible fluid flow
  - Advective-diffusive transport

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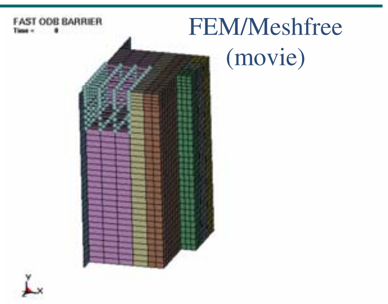
Crashworthiness: ODB Model

LS-DYNA



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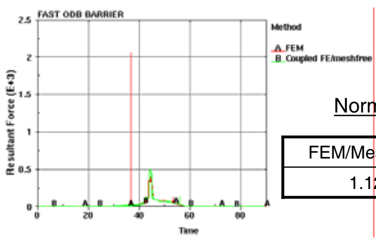

Normalized CPU Time

FEM/Meshfree	FEM
1.12	1.00

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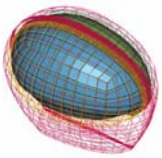
Brain Injury Simulation **LS-DYNA**

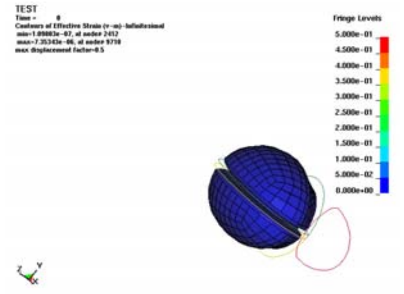
FEM/Meshfree(discretization)

FEM/Meshfree(movie)

Visco-hyperelastic Material



DOT/NHTSA
SIMon model



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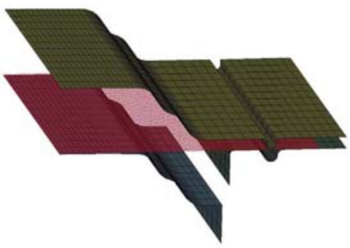
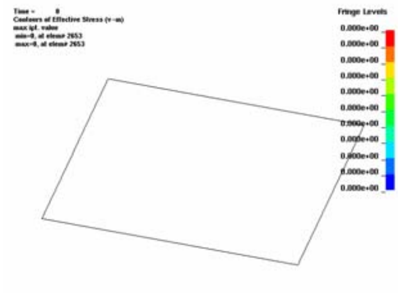
Channel Forming Simulation (Explicit) **LS-DYNA**

Meshfree(discretization)



EFG nodes 19026



Meshfree(movie)

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Recent Developments of LS-DYNA-EFG

LS-DYNA

- Inclusion of Different EFG Background Elements
- Parallel Mesh-free Computation
- Implicit Mesh-free Shells

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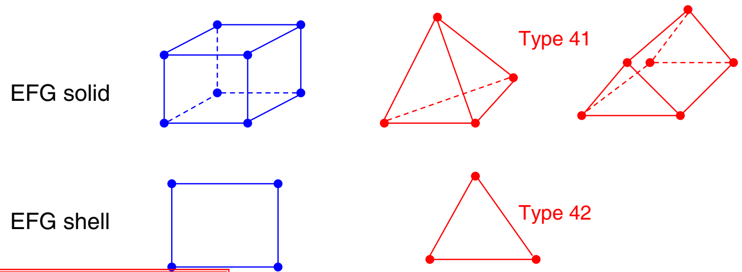
Inclusion of Different EFG Background Element

LS-DYNA

- **Tetrahedron Element in FEM**
  1. 4-noded constant stress (#10)
  2. 10-noded 5-stress points (#16)
  3. 4-noded nodal pressure for bulk forming(#13)

- **Inclusion of Background Element in EFG**



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Inclusion of Different EFG Background Element (cont.) LS-DYNA

Tetrahedron Background Element in EFG

Lagrangian Strain Smoothing + Assumed Strain Method

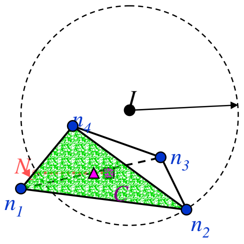
$$\Delta\tilde{u}_{i,j}(X_L)=\left[\sum_I \int_{\Gamma_L^X} \Psi_I N_k d\Gamma \Delta d_{iI}\right]\tilde{F}_{kj}^{-1}(X_L)$$

$$\delta\Pi(u,\tilde{F}) = \int_\Omega \delta\tilde{F}_{ik}\tilde{F}_{kj}^{-1}\tilde{\tau}_{ij}(\tilde{F})\tilde{J}^0(\tilde{F})d\Omega - \delta W^{ext}(u)$$

$$f_I^{int} = \sum_{L=1}^{NP}\tilde{B}_I^T(X_L)\tilde{G}^T(X_L)\tilde{\tau}(\tilde{F}(X_L))\tilde{J}^0(X_L)A_L$$

Second-order accuracy for cell-averaged data
[Frink et al. 1994]

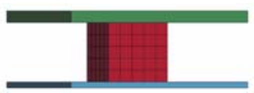
$$\Psi_I(X_{F(124)}) = \Psi_I(X_c) + \frac{1}{4}\left[\frac{1}{3}(\Psi_I(X_{N1})+\Psi_I(X_{N2})+\Psi_I(X_{N4}))-\Psi_I(X_{N3})\right]$$



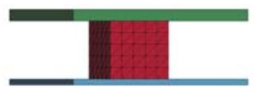
1. 4-noded (#41) in ls971 — • non-corotational foam materials
2. 4-noded in nearly incompressible limit (?) *Requires research !*

Foam Material Simulation LS-DYNA

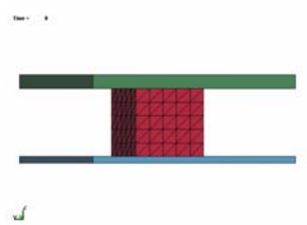
Low-density foam



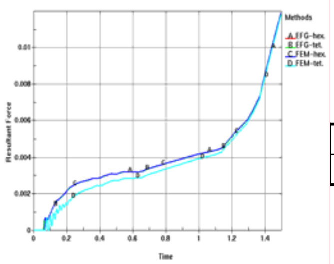
Hexahedron Element



Tetrahedron Element



EFG-tet.(movie)

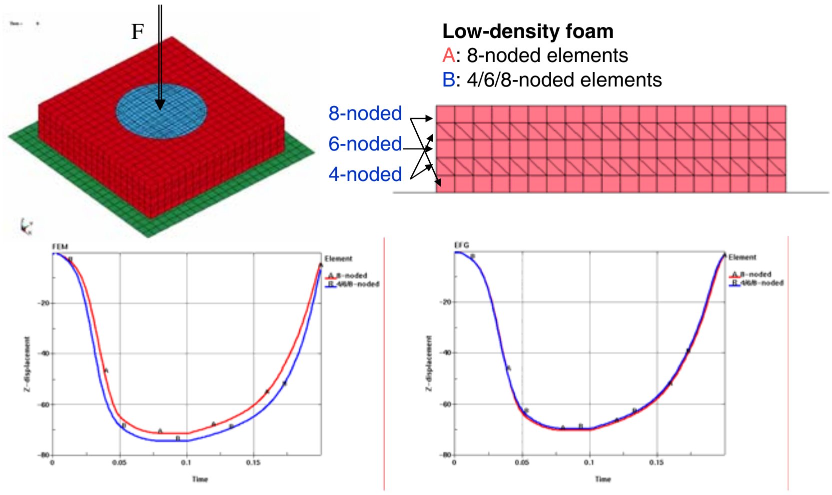


Compressibility

FEM RI-hex.#1	FEM-tet.#10	EFG-hex.#41	EFG-tet.#41
95 %	95 %	95 %	95 %

Compression Test

LS-DYNA



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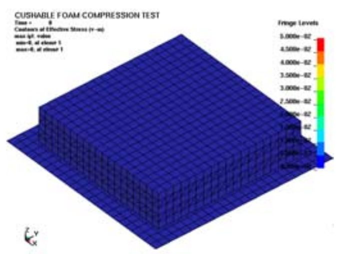
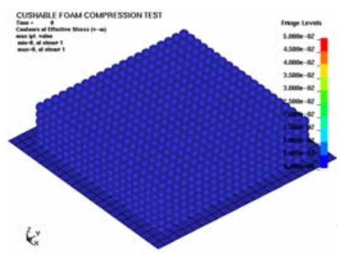
Compression Test (cont.)

LS-DYNA



FEM(movie)



Meshfree(movie)

Normalized CPU Time

FEM RI	EFG
1.0	5.4

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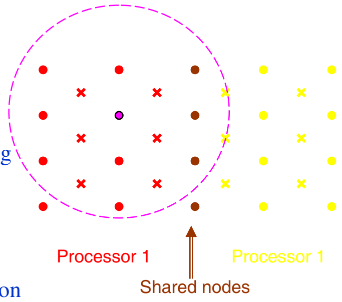
Parallel Mesh-free Computation

LS-DYNA

- SMP
  - Scalability depends on the efficiency of the matrix multiplication with the transformation matrix
- MPP
  - Difficulty computing A inverse in MPP
  - More communications in a EFG region among processors
    - ✓ More neighbors involve in nodal force computation
    - ✓ Another set of neighbors in transformation



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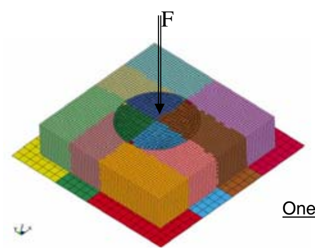
Compression Test

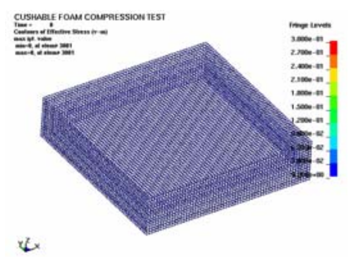
LS-DYNA



Crushable Foam Material

Total 53624 EFG nodes
SGI ORIGIN 2000 16-CPU 500 MHz

One EFG part on 8 processors



Normalized SMP and MPP CPU Time

No. of CPU's	1	2	4	8
SMP	1.00	0.55	0.43	0.32
MPP	0.99	0.52	0.25	0.15

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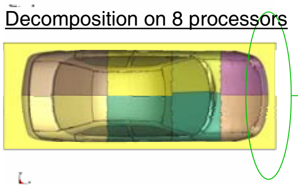
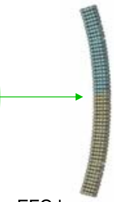
Front Impact of a Neon Car

LS-DYNA

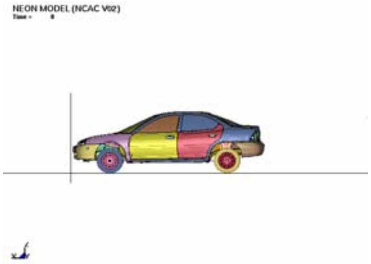
Decomposition on 8 processors

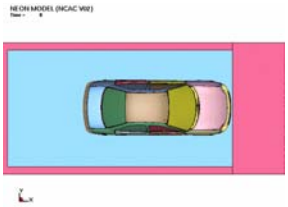




One EFG bumper on 2 processors

Total nodes 285879
Total solid elements 2969
Total shell elements 269107
EFG solid: foam bumper 1200
SGI ORIGIN 2000 16-CPU 500 MHz



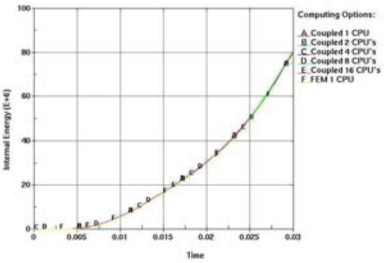


Front Impact of a Neon Car (cont.)

LS-DYNA

IA-64 HP-UX Clusters
rx2600 (2u 2p)
GigE (IB Q2'04)
ClusterPak

Comparison of Internal Energy



HP Benchmarking Systems
Clusters of HP RX2600 (2 1.5 MHz CPU's per node)
OS: HP-UX 11.23
LS-DYNA: double precision MPP 971.2875
Thanks to Dr. Yih-Yih Lin of HP

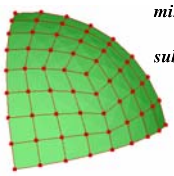
Normalized MPP CPU Time

No. of CPU's	1	2	4	8	16
Coupled FEM/EFG	1.00	0.71	0.36	0.22	0.15

Development of Meshfree Shell [Wu and Guo 2002] LS-DYNA

- Global Approach

Angle-based triangular flattening (Sheffer and Sturler, 2001)+ Moving least-squares

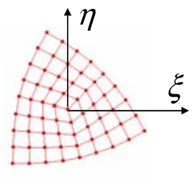


$$\textbf{minimize } F(\alpha)=\sum_{i=1}^{N}\sum_{j=1}^{3}(\alpha_i^j-\varphi_i^j)^2 w_i^j$$

$$\textbf{subject to } g_{i,j}^{(1)} \equiv \alpha_i^j \ge \varepsilon_2 > 0, \text{ for } i=1...N,\ j=1...3, \text{ and some } \varepsilon_2 > 0$$

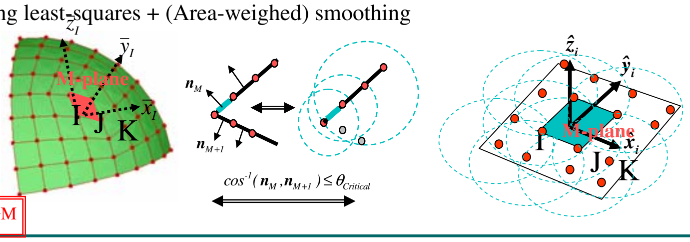
⋮

Projection



- Local Approach

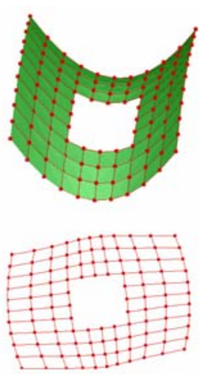
Moving least-squares + (Area-weighed) smoothing



Supported by GM

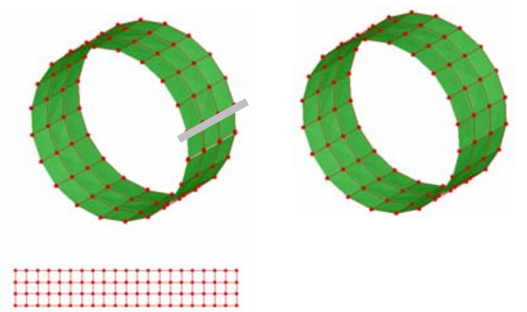
Constructed Meshfree Surface LS-DYNA

Meshfree Global Approach
Meshfree Local Approach



Meshfree Global Approach
Meshfree Local Approach

Meshfree Local Approach



Meshfree Shell Formulation

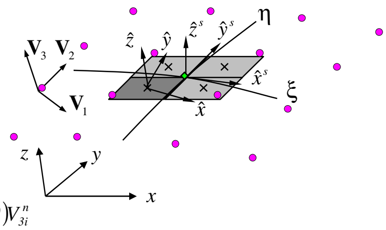
LS-DYNA

- First-order shear deformable shell theory with 5/6 –parameter approach

$$\mathcal{B} := \left\{ \boldsymbol{x} \in \mathbb{R}^3 \middle| \boldsymbol{x}(\xi,\eta,\zeta,t) = \phi(\xi,\eta,t) + \zeta \boldsymbol{V}_3(\xi,\eta,t) \;\; with\; \zeta \in \left[-\frac{h}{2}, +\frac{h}{2} \right] \right\}$$

$$\Delta \boldsymbol{V}_3 = -\boldsymbol{V}_2 \Delta\alpha + \boldsymbol{V}_1 \Delta\beta$$

- A co-rotational coordinate system is embedded at each in-plane integration point and defined by the convected coordinates



Two approximations for local velocity

$$\hat{v}_i = \sum_{I=1}^{NP} \tilde{\Psi}_I \hat{v}_{iI} + \zeta \sum_{I=1}^{NP} \tilde{\Psi}_I \frac{t_I}{2} \left[-V_{2iI} \quad V_{1iI} \right] \begin{Bmatrix} \dot{\alpha}_I \\ \dot{\beta}_I \end{Bmatrix}; \; V_{3i}^{n+1} = R_{ij}(\Delta\theta) V_{3i}^{n}$$

$$\hat{v}_i = \sum_{I=1}^{NP} \tilde{\Psi}_I \hat{v}_{iI} + \zeta \sum_{I=1}^{NP} \tilde{\Psi}_I \frac{t_I}{2} \begin{Bmatrix} \dot{\hat{\theta}}_{xI} \\ \dot{\hat{\theta}}_{yI} \\ 0 \end{Bmatrix} \qquad (|v_3 \cdot \hat{z}| < 0.01)$$

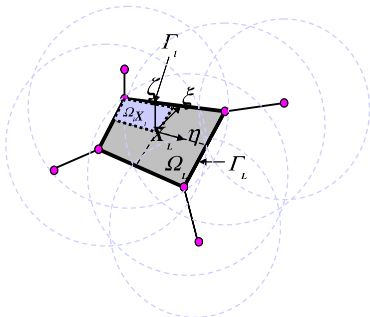
Meshfree Shell Formulation (cont.)

LS-DYNA

- Lagrangian smoothed strains in co-rotational system [Chen and Wu 1998]

$$\tilde{\boldsymbol{\varepsilon}}^m = \sum_I \tilde{\boldsymbol{B}}_I^m \boldsymbol{d}_I \qquad \tilde{\boldsymbol{\varepsilon}}^b = \zeta \sum_I \tilde{\boldsymbol{B}}_I^b \boldsymbol{d}_I \qquad \tilde{\boldsymbol{\varepsilon}}^s = \sum_I \tilde{\boldsymbol{B}}_I^s \boldsymbol{d}_I$$

$$\begin{cases} \tilde{\boldsymbol{B}}_I^m(\boldsymbol{x}_l) = \dfrac{1}{A_l} \displaystyle\int_{\Gamma_l} \hat{\boldsymbol{B}}_I^m \cdot \boldsymbol{n} d\Gamma \\ \tilde{\boldsymbol{B}}_I^b(\boldsymbol{x}_l) = \dfrac{1}{A_l} \displaystyle\int_{\Gamma_l} \hat{\boldsymbol{B}}_I^b \cdot \boldsymbol{n} d\Gamma \\ \tilde{\boldsymbol{B}}_I^s(\boldsymbol{x}_L) = \dfrac{1}{A_L} \displaystyle\int_{\Gamma_L} \hat{\boldsymbol{B}}_I^s \cdot \boldsymbol{n} d\Gamma \end{cases} \quad \text{where} \quad \begin{cases} \displaystyle\sum_{J=1} \tilde{\nabla} \tilde{\Psi}_I(\boldsymbol{X}) W_J = \boldsymbol{0} \\ \begin{cases} \displaystyle\sum_{I=1} \tilde{\nabla} \tilde{\Psi}_I(\boldsymbol{X}) = \boldsymbol{0} \\ \displaystyle\sum_{I=1} \tilde{\nabla} \tilde{\Psi}_I(\boldsymbol{X}) X_{iI}^2 = X_i \end{cases} \end{cases}$$



- Internal nodal force

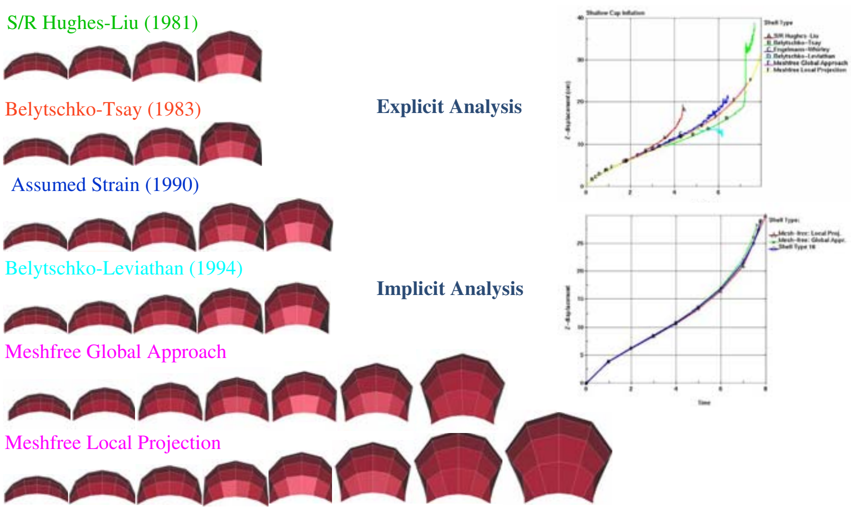
$$\boldsymbol{f}_I^{int} = \int_\Omega \tilde{\boldsymbol{B}}_I^{m^T} \cdot \boldsymbol{\Phi} \cdot \hat{\boldsymbol{\sigma}} d\Omega + \int_\Omega \zeta \tilde{\boldsymbol{B}}_I^{b^T} \cdot \boldsymbol{\Phi} \cdot \hat{\boldsymbol{\sigma}} d\Omega + \int_\Omega \tilde{\boldsymbol{B}}_I^{s^T} \cdot \boldsymbol{\Phi} \cdot \hat{\boldsymbol{\sigma}} d\Omega$$

Summary

- thin to moderate thick shell
- Show advantages in membrane and bending-dominant problems

Clamped Shallow Cap Under Inflation

LS-DYNA



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Spring-back Analysis (Explicit-Implicit Switch) : NUMISHEET 93

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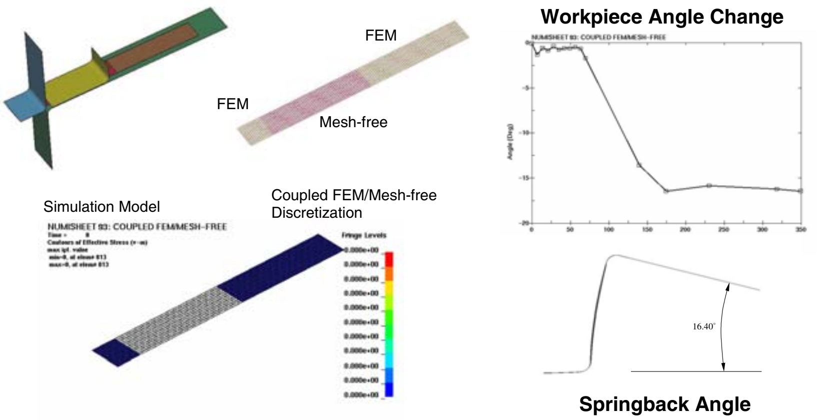

Springback Angle

Meshfree	Experiment
16.4	17.1

Coupled FEM/Mesh-free (movie)

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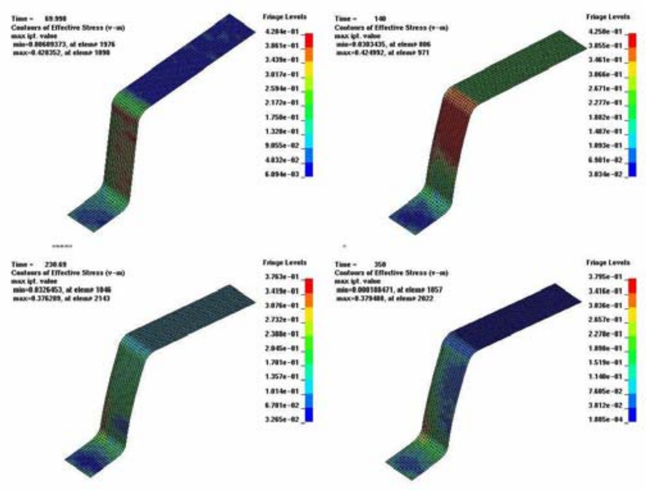
Effective Stress during Springback



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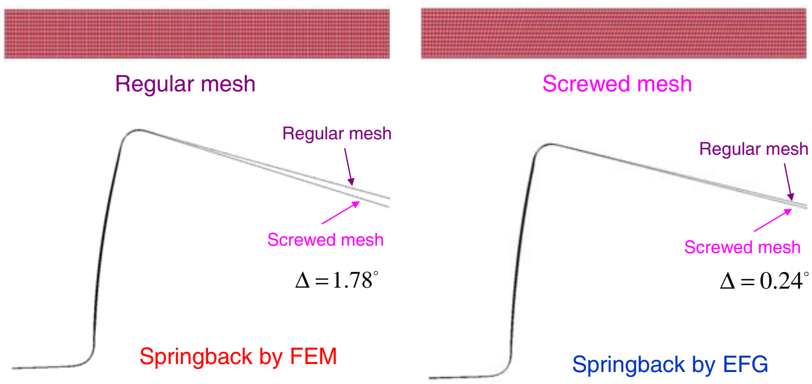
Springback Analysis: Disretization Effect



Required Improvements and Main Issues in EFG Analysis

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- **Methodology**
  - Material separation
  - Mixed formulation in nearly incompressible limit
- Discretization and Approximation
- Numerical Integration
  - Spatial integration
    - Inclusion of tetrahedron and pentahedron background elements
  - Time integration
- Objective or **multiplicative Stress Update**
- Others
  - **Contact algorithm**
  - Parallilization
    - SMP, MPP, Optimized

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On-going and Future Plans

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2004 LS-DYNA EFG Update

	2D & 3D SMP	Mix formulation	Inclusion of 4/6/8-noded elements	Solid MPP	Two explicit EFG shells
ls970	√	√	√		
ls971	√	√	√	√	√

	Fast EFG method	3D EFG material erosion	Thickness stress for shell
ls970			
ls971	√	√	√

Future Plans

- Discrete and continuous fracture analysis in mesh-free solid and shell
- Coupling systems
- Mesh-free based contact algorithm
- 3D Interactive adaptivity
- Fluid and Gas formulations

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