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Computational Micro-Mechanical Model of Composite & Flexible Woven Fabric with Fiber Reorientation

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OUTLINE

- Motivation of the model development
- State of the problem
- Goals of the model development
- Micro-mechanical model
- Homogenization procedure
- Fiber reorientation
- Material nonlinearity
- Failure model
- Numerical examples
- Impact simulations
- Conclusions

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MOTIVATION OF THE MODEL

- Composite materials have high energy absorption capability in impact and therefore they can be used as crashworthiness materials in vehicle structures or even as armors to protect some parts.
- Crash and impact experiments for structure survivability estimation are very expensive compared to the computer impact simulations, using finite element code for dynamic problems with explicit time integration.

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STATE OF THE PROBLEM

- **Woven fabric composites** have complex structure and sophisticated micro-mechanical models are necessary to predict their elastic properties in all directions
- They render different nonlinear behavior in the different loading directions
- The material nonlinearity of the matrix is combined with the scissoring effect of the fiber reorientation
- Failure analysis and the failure modeling are is still being investigated because of the complex structure and the complex interaction of the components

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STATE OF THE PROBLEM

(Continued)

- Most of the successful micro-mechanical material models of woven fabric composites have high discretization of their Representative Unit Volume which leads to high computational costs
- Contrary, FE impact simulations need computationally efficient material models because of the repeated calculations of the model at each inherent small time step of the simulation

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GOALS OF THE MODEL DEVELOPMENT

- Development of woven fabric composite material model based on micro-mechanical approach and Representative Volume Cell (RVC) approach
- The RVC has to represent the pattern of the deformed composite material in order to account for fiber reorientation
- Geometrical nonlinearity due to fiber reorientation and material nonlinearity due to the matrix material shear have to be included
- Adequate and efficient failure model based on micro-mechanical failure criteria
- Computationally efficient model with simplified geometry in order to avoid high discretization of the RVC

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MICRO-MECANICAL MODEL

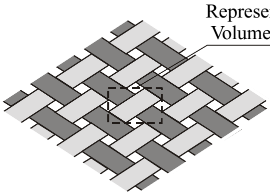
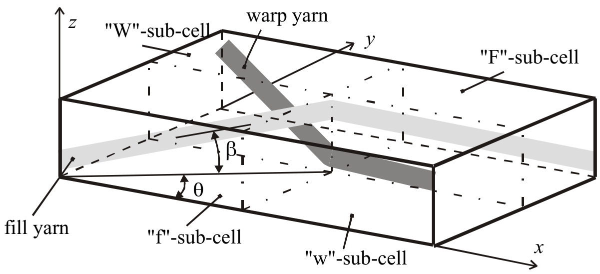


Fig. 1. Woven composite interlacing pattern.

Fig. 2. Micro-mechanical model of RVC.



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MICRO-MECANICAL MODEL

(Continued)

Discount factors are used to degrade the material stiffness $d_i \in (0,1]$

The stiffness matrix of the yarn has to obey the ratio $\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$, $i, j = 1,2,3$

$$[C]_y = [S]_y^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\sqrt{\frac{\nu_{12}}{E_1}\frac{\nu_{21}}{d_2E_2}} & -\sqrt{\frac{\nu_{12}}{E_1}\frac{\nu_{21}}{d_3E_2}} & 0 & 0 & 0 \\ -\sqrt{\frac{\nu_{12}}{E_1}\frac{\nu_{21}}{d_2E_2}} & \frac{1}{d_2E_2} & -\frac{\nu_{23}}{\sqrt{d_2E_2d_3E_2}} & 0 & 0 & 0 \\ -\sqrt{\frac{\nu_{12}}{E_1}\frac{\nu_{21}}{d_3E_2}} & -\frac{\nu_{23}}{\sqrt{d_2E_2d_3E_2}} & \frac{1}{d_3E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{d_4G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{d_5G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{d_6G_{12}} \end{bmatrix}^{-1}$$

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MICRO-MECANICAL MODEL

(Continued)

The stiffness matrix of the matrix material

$$[C]_m = [S]_m^{-1} = \begin{bmatrix} \frac{1}{d_E E} & -\frac{\nu}{d_E E} & -\frac{\nu}{d_E E} & 0 & 0 & 0 \\ -\frac{\nu}{d_E E} & \frac{1}{d_E E} & -\frac{\nu}{d_E E} & 0 & 0 & 0 \\ -\frac{\nu}{d_E E} & -\frac{\nu}{d_E E} & \frac{1}{d_E E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{d_G G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{d_G G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{d_G G} \end{bmatrix}^{-1}$$

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MICRO-MECANICAL MODEL

(Continued)

Two levels of elastic property homogenization:

I. Homogenization of yarn and matrix materials to obtain the stiffness of the sub-cell.

II. Homogenization of sub-cells to obtain the stiffness of RVC.

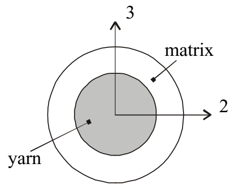

Fig. 3. I level homogenization model.

Iso-strain and iso-stress boundary conditions at the I-st level of homogenization

$\varepsilon_1^y = \varepsilon_1^m$, $\varepsilon_4^y = \varepsilon_4^m$, $\varepsilon_6^y = \varepsilon_6^m$,

$\sigma_2^y = \sigma_2^m$, $\sigma_3^y = \sigma_3^m$, $\sigma_5^y = \sigma_5^m$.

The contracted notation accepted here is

$1 \cong 11, 2 \cong 22, 3 \cong 33, 4 \cong 12, 5 \cong 23, 6 \cong 31$

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MICRO-MECANICAL MODEL

(Continued)

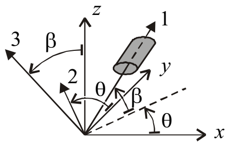

Fig. 4. Yarn orientation

Sub-cell stiffness transformation $[C]=[T]^{\mathrm{T}}[C'][T]$

The transformation matrix $[T]=\begin{bmatrix}[T_1] & [T_2]\\ [T_3] & [T_4]\end{bmatrix}$

Iso-strain and iso-stress boundary conditions at the II-nd level of homogenization

$$\varepsilon_1^f=\varepsilon_1^w=\varepsilon_1^F=\varepsilon_1^W\ ,\quad \varepsilon_2^f=\varepsilon_2^w=\varepsilon_2^F=\varepsilon_2^W\ ,\quad \varepsilon_4^f=\varepsilon_4^w=\varepsilon_4^F=\varepsilon_4^W\ ,$$
$$\sigma_3^f=\sigma_3^w=\sigma_3^F=\sigma_3^W\ ,\quad \sigma_5^f=\sigma_5^w=\sigma_5^F=\sigma_5^W\ ,\quad \sigma_6^f=\sigma_6^w=\sigma_6^F=\sigma_6^W\ .$$

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HOMOGENIZATION PROCEDURE

Strain and stress vectors as well as stiffness matrices of constituents are partitioned into iso-strain and iso-stress components.

$$\{\varepsilon\}_k=\left\{\{\varepsilon_n\}_k^{\mathrm{T}}\ \ \{\varepsilon_s\}_k^{\mathrm{T}}\right\}_k^{\mathrm{T}}\qquad \{\sigma\}_k=\left\{\{\sigma_n\}_k^{\mathrm{T}}\ \ \{\sigma_s\}_k^{\mathrm{T}}\right\}_k^{\mathrm{T}}\qquad [C]_k=\begin{bmatrix}[C_{nn}]_k & [C_{ns}]_k\\ [C_{sn}]_k & [C_{ss}]_k\end{bmatrix}$$

The constitutive equations for constituents can be written

$$\{\sigma_n\}_k=[C_{nn}]_k\{\varepsilon_n\}_k+[C_{ns}]_k\{\varepsilon_s\}_k\qquad \{\sigma_s\}_k=[C_{sn}]_k\{\varepsilon_n\}_k+[C_{ss}]_k\{\varepsilon_s\}_k$$

The effective strain and stress are volumetric average of the constituents strain and stress, respectively. The effective stiffness matrix is the ultimate aim.

$$\{\bar{\varepsilon}\}=\sum_{k=1}^{N} f_k\{\varepsilon\}_k\qquad \{\bar{\sigma}\}=\sum_{k=1}^{N} f_k\{\sigma\}_k\qquad \{\bar{\sigma}\}=[\bar{C}]\{\bar{\varepsilon}\}$$

Partitioning the effective stiffness matrix, the constitutive equations can be written

$$\{\bar{\sigma}_n\}=[\bar{C}_{nn}]\{\bar{\varepsilon}_n\}+[\bar{C}_{ns}]\{\bar{\varepsilon}_s\}\quad \{\bar{\sigma}_s\}=[\bar{C}_{sn}]\{\bar{\varepsilon}_n\}+[\bar{C}_{ss}]\{\bar{\varepsilon}_s\}\quad [\bar{C}]=\begin{bmatrix}[\bar{C}_{nn}] & [\bar{C}_{ns}]\\ [\bar{C}_{sn}] & [\bar{C}_{ss}]\end{bmatrix}$$

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HOMOGENIZATION PROCEDURE

(Continued)

Applying the mixed boundary conditions $\{\overline{\sigma}_s\}=\{\sigma_s\}_k \quad \{\overline{\varepsilon}_n\}=\{\varepsilon_n\}_k$

$$\{\varepsilon_s\}_k=[C_{ss}]_k^{-1}\{\overline{\sigma}_s\}-[C_{ss}]_k^{-1}[C_{sn}]_k\{\overline{\varepsilon}_n\}$$

$$\{\sigma_n\}_k=\left([C_{nn}]_k-[C_{ns}]_k[C_{ss}]_k^{-1}[C_{sn}]_k\right)\{\overline{\varepsilon}_n\}+[C_{ns}]_k[C_{ss}]_k^{-1}\{\overline{\sigma}_s\}$$

Applying the rule of mixture $\{\overline{\sigma}_n\}=\sum_{k=1}^{N} f_k\{\sigma_n\}_k \quad \{\overline{\varepsilon}_s\}=\sum_{k=1}^{N} f_k\{\varepsilon_s\}_k$

$$[\overline{\sigma}_n]=[C_1^*]\{\overline{\varepsilon}_n\}+[C_2^*]\{\overline{\sigma}_s\} \qquad [\overline{\varepsilon}_s]=[C_3^*]\{\overline{\sigma}_s\}-[C_4^*]\{\overline{\varepsilon}_n\}$$

where $[C_1^*]=\sum_{k=1}^{N} f_k\left([C_{nn}]_k-[C_{ns}]_k[C_{ss}]_k^{-1}[C_{sn}]_k\right) \quad [C_2^*]=\sum_{k=1}^{N} f_k[C_{ns}]_k[C_{ss}]_k^{-1}$

$$[C_3^*]=\sum_{k=1}^{N} f_k[C_{ss}]_k^{-1} \qquad [C_4^*]=\sum_{k=1}^{N} f_k[C_{ss}]_k^{-1}[C_{sn}]_k$$

The constitutive equations for effective strain and stress can be written

$$\{\overline{\sigma}_n\}=\left([C_1^*]+[C_2^*][C_3^*]^{-1}[C_4^*]\right)\{\overline{\varepsilon}_n\}+[C_2^*][C_3^*]^{-1}\{\overline{\varepsilon}_s\}$$

$$\{\overline{\sigma}_s\}=[C_3^*]^{-1}[C_4^*]\{\overline{\varepsilon}_n\}+[C_3^*]^{-1}\{\overline{\varepsilon}_s\}$$

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HOMOGENIZATION PROCEDURE

(Continued)

The effective stiffness matrix finally is obtained

$$[\overline{C}_{nn}]=[C_1^*]+[C_2^*][C_3^*]^{-1}[C_4^*] \qquad [\overline{C}_{ns}]=[C_2^*][C_3^*]^{-1}$$

$$[\overline{C}_{sn}]=[C_3^*]^{-1}[C_4^*]=[\overline{C}_{ns}]^{T} \qquad [\overline{C}_{ss}]=[C_3^*]^{-1}$$

As a generalization of the homogenization procedure, it can be stipulated in three steps:

- Choosing iso-strain and iso-stress components and partitioning the constituent stiffness matrices
- Calculating the interim matrices denoted by star-superscript
- Calculating the partitions of the effective stiffness matrix

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FIBER REORIENTATION

Unit directional fiber vectors are formulated for fill and warp yarns

$$\{q_f\}=\{\cos\beta_f\cos\theta_f \quad \cos\beta_f\sin\theta_f \quad \sin\beta_f\}^T$$

$$\{q_w\}=\{\cos\beta_w\cos\theta_w \quad \cos\beta_w\sin\theta_w \quad \sin\beta_w\}^T$$

They are rotated by the deformation gradient tensor and then normalized

$$\{q'_f\}=[F]\{q_f\}, \quad \{q'_w\}=[F]\{q_w\} \qquad \{q_f\}=\{q'_f\}/\|\{q'_f\}\|, \quad \{q_w\}=\{q'_w\}/\|\{q'_w\}\|$$

where the approximated tensor $$[F]=\begin{bmatrix} 1+d\bar{\varepsilon}_1 & \frac{d\bar{\varepsilon}_4}{2} & \frac{d\bar{\varepsilon}_6}{2} \\ \frac{d\bar{\varepsilon}_4}{2} & 1+d\bar{\varepsilon}_2 & \frac{d\bar{\varepsilon}_5}{2} \\ \frac{d\bar{\varepsilon}_6}{2} & \frac{d\bar{\varepsilon}_5}{2} & 1+d\bar{\varepsilon}_3 \end{bmatrix}$$

Now, the updated orientation angles of the yarns are obtained

$$\beta_f=\sin^{-1}q_{f3}, \quad \beta_w=\sin^{-1}q_{w3} \quad \theta_f=\tan^{-1}(q_{f2}/q_{f1}), \quad \theta_w=\tan^{-1}(q_{w2}/q_{w1})$$

Initially, $\beta_f=\beta_w=\beta_0$, $\theta_f=45°$, $\theta_w=-45°$

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MATERIAL NONLINEARITY

Material nonlinearity is described by the Romberg-Osgood equation

$$\tau(\gamma)=\frac{G_0\gamma}{\left[1+\left(\frac{G_0\gamma}{S}\right)^p\right]^{\frac{1}{p}}}$$

The tangential shear modulus can be determined as a function of the shear stress instead of the shear strain

$$G_t=\frac{d\tau}{d\gamma} \qquad \gamma(\tau)=\frac{\tau}{G_0\left[1-\left(\frac{\tau}{S}\right)^p\right]^{\frac{1}{p}}}$$

$$G_t=\frac{1}{\frac{d\gamma}{d\tau}}=G_0\left[1-\left(\frac{\tau}{S}\right)^p\right]^{1+\frac{1}{p}}$$

The material nonlinearity can be introduced as a discount factor of shear moduli $d_s=\frac{G_t}{G_0}$

$$d_{sG}=\left[1-\left(\sqrt{\frac{3}{2}}\frac{\tau_o}{S}\right)^{p_m}\right]^{1+\frac{1}{p_m}}$$

$$d_{s4}=\left[1-\left(\frac{\sigma_4^y}{S_t}\right)^{p_y}\right]^{1+\frac{1}{p_y}}, \quad d_{s5}=\left[1-\left(\frac{\sigma_5^y}{S_t}\right)^{p_y}\right]^{1+\frac{1}{p_y}}, \quad d_{s6}=\left[1-\left(\frac{\sigma_6^y}{S_l}\right)^{p_y}\right]^{1+\frac{1}{p_y}}$$

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FAILURE MODEL

Failure mode	Failure condition	Discount coefficients				
		d_2	d_3	d_{f4}	d_{f5}	d_{f6}
Longitudinal tension	$c_t \sigma_1^y > X_t$	fiber breakage - ultimate failure				
Longitudinal compression	$-c_c \sigma_1^y > X_c$	fiber breakage - ultimate failure				
Transverse tension, 2-direction	$\sigma_2^y > Y_t$	0.01	1.00	0.20	1.00	0.20
Transverse compression, 2-direction	$-\sigma_2^y > Y_c$	0.01	1.00	0.20	1.00	0.20
Transverse tension, 3-direction	$\sigma_3^y > Y_t$	1.00	0.01	0.20	1.00	0.20
Transverse compression, 3-direction	$-\sigma_3^y > Y_c$	1.00	0.01	0.20	1.00	0.20
Longitudinal shear, 12-plane	$\lvert\sigma_4^y\rvert > S_l$	0.01	1.00	0.01	1.00	1.00
Transverse shear, 23-plane	$\lvert\sigma_5^y\rvert > S_t$	0.01	0.01	0.01	0.01	0.01
Longitudinal shear, 31-plane	$\lvert\sigma_6^y\rvert > S_l$	1.00	0.01	1.00	1.00	0.01

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FAILURE MODEL

(Continued)

For the matrix material, the maximal principle stress is the failure criterion

If $\max\{\sigma_I^m, \sigma_{II}^m, \sigma_{III}^m\} > X_m$ then $d_E = 0.01$, $d_{fG} = 0.20$

The minimum of the discount factors from material nonlinearity and from failure model is used for shear moduli stiffness matrix degradation:

For matrix material stiffness matrix $d_G = \min\{d_{sG}, d_{fG}\}$

For yarn material stiffness matrix

$$d_4 = \min\{d_{s4}, d_{f4}\}, \quad d_5 = \min\{d_{s5}, d_{f5}\}, \quad d_6 = \min\{d_{s6}, d_{f6}\}$$

The total strain of the RVC is accumulated at each time step. If the maximal principle strain or the maximal shear strain of the RVC exceeds the ultimate strain for the integrity, E_u, a ultimate failure is accounted for the material model.

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NUMERICAL EXAMPLES

The Graphite/Epoxy material AS4/3501-6 with properties described by Blackketter et. al. is used for validation.

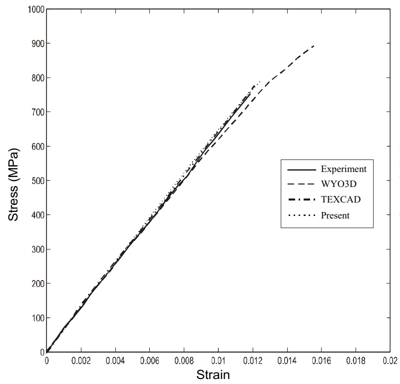

Fig. 5. 0/90 deg. tension.

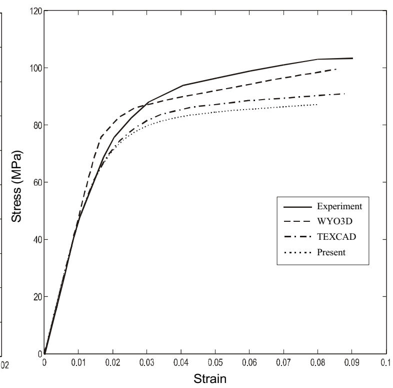

Fig. 6. 0/90 deg. shear.

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NUMERICAL EXAMPLES

(Continued)

The Graphite/Epoxy IM7/8551 7A 5-harness satin material with properties described by Karayaka and Kurath

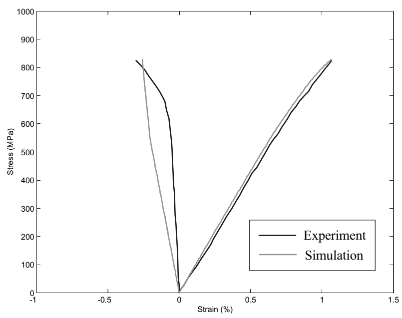

Fig. 7. 0/90 deg. tension.

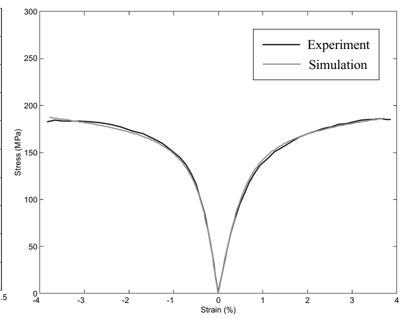

Fig. 8. +45/-45 deg. tension.

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NUMERICAL EXAMPLES

(Continued)

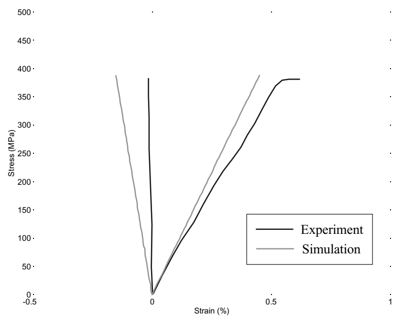

Fig. 9. 0/90 deg. compression.

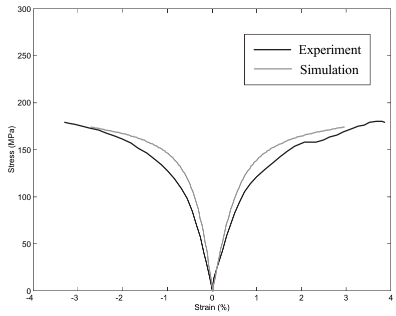

Fig. 10. +45/-45 deg. compression.

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NUMERICAL EXAMPLES

(Continued)

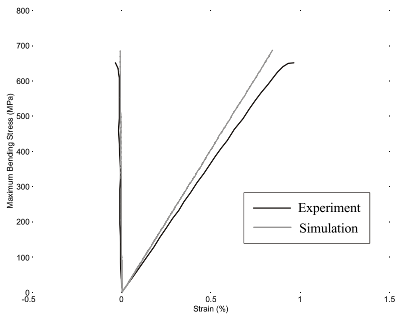

Fig. 11. 0/90 deg. bending.

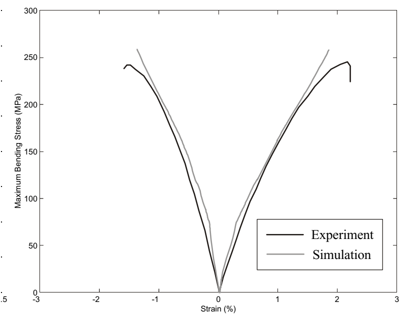

Fig. 12. +45/-45 deg. bending.

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IMPACT SIMULATIONS

Impact penetration of 0.5 in bullet with 2000 m/s velocity into Eglass/epoxy composite armor.

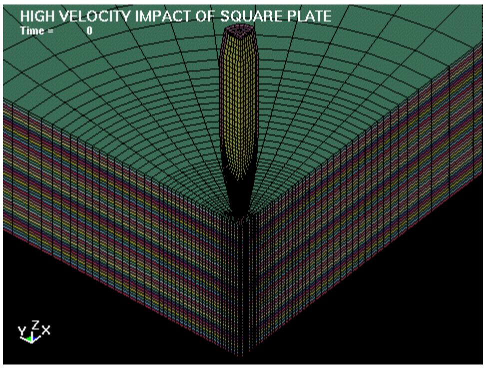

Fig. 13. Impact penetration, Johnson-Cook material model and user defined woven fabric material model with eroding contact.

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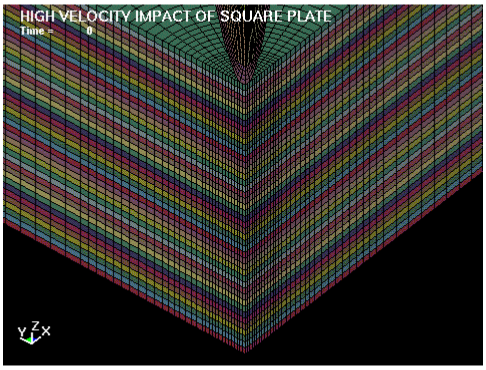
IMPACT SIMULATIONS

(Continued)



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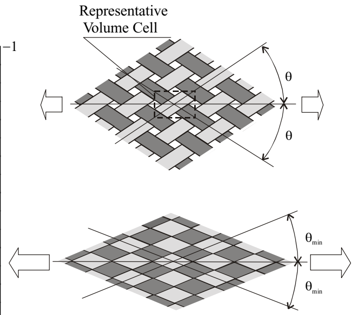
Flexible Woven Fabric

- The direction of the yarn in each sub-cell is determined by two angles – the braid angle, θ, and the undulation angle of the yarn, which is different for the fill and warp-yarns, β_f and β_w, respectively.
- The starting point for the homogenization of the material properties is the determination of the yarn stiffness matrices. The material of the yarn is assumed to be transversely isotropic

$$[C]=[S]^{-1}=\begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\mu G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\mu G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\mu G_{12}} \end{bmatrix}^{-1}$$



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- μ is a discount factor, which is a function of the braid angle, θ, and has value between μ_0 and 1. Initially, in free stress state, the magnitude of the discount factor is very small (μ_0 << 1) and the material has very small resistance to shear deformation.
- When locking occurs, the fabric yarns are packed and they behave like an elastic media. The discount factor is unity in this case and the fabric material resists the shear deformation with its real shear moduli.
- The discount factor, μ, is a function of the braid angle and it has to switch the model from trellis mechanism to elastic media and vise versa. A piece-wise function with two constants is chosen for the discount factor as follows:

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$$\mu = \begin{cases} 1, & \theta < \theta_{dn} \\ \mu_0, & \theta \in (\theta_{dn}, \theta_{up}) \\ 1, & \theta > \theta_{up} \end{cases}$$

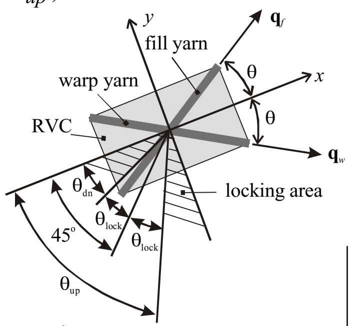
the lowest locking angle,

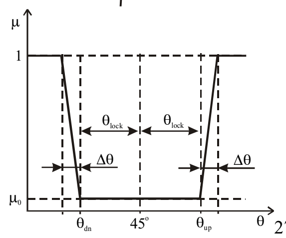
$$\theta_{dn} = 45° - \theta_{lock}$$

$$\theta_{up} = 45° + \theta_{lock}$$

is the highest locking angle

θ_{lock} is the range to the locking angles





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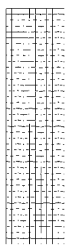

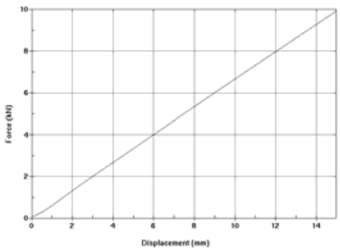

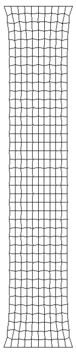

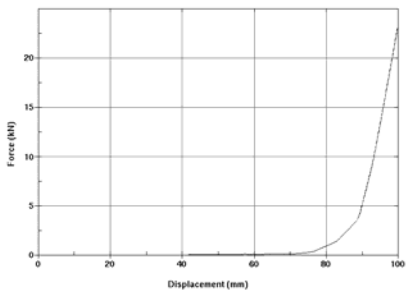

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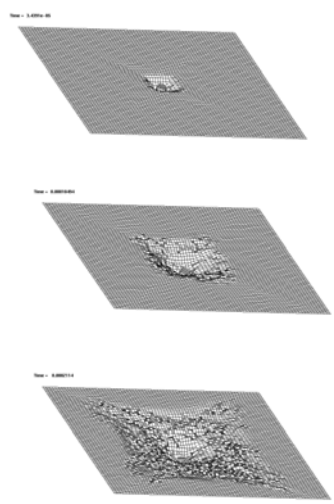

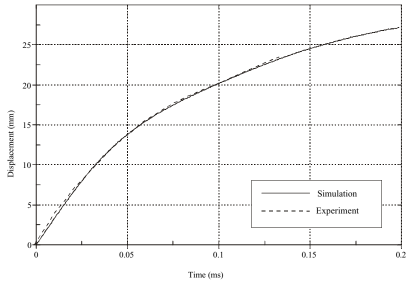

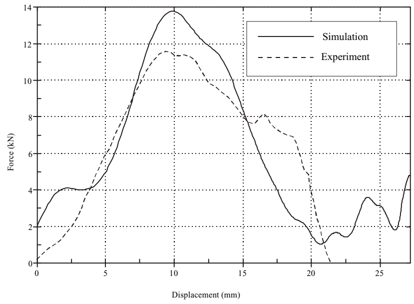

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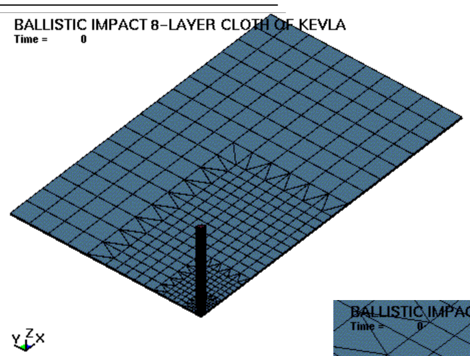

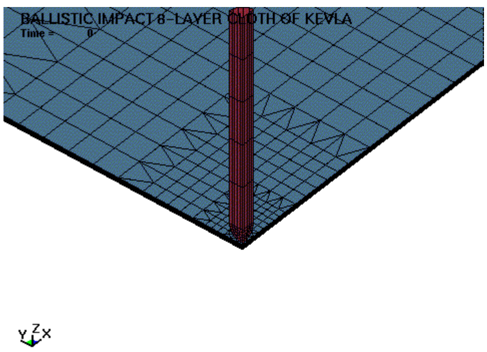

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CONCLUSIONS

- The stress response of the developed micro-mechanical material model of woven fabric composites is in good agreement with the experiments for various loadings.
- The model is implemented in LS-DYNA FE code as a user defined material model and it shows computational efficiency and a potential for large-scale simulations.
- The flexible fabric material model yield excellent prediction relative to experiments.
- The model is appropriate for FE impact simulations and for structure survivability estimation.

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