Recent developments in the Electromagnetic Module: a new 2D axi-symmetric EM solver

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1 Introduction

An electromagnetism module has being developed in LS-DYNA for coupled mechanical/thermal/electromagnetic simulations. More recently, a new 2D axi-symmetric version of this solver was introduced, allowing much faster simulations.
In this solver, the EM equations are solved in a 2D plane, and the 2D EM fields, Lorentz force and Joule heating are then expanded to 3D elements by rotations around the axis. This allows to couple the 2D EM with 3D mechanics and thermal, thus keeping all the LS-DYNA 3D capabilities available. The user needs to provide a 3D mesh with rotational symmetry, either on the full 360 degrees or a small slice.
The 2D-EM eddy-current problems are solved using a coupled FEM-BEM method, both based on differential forms. It can be coupled to different external circuits, including imposed currents, imposed voltage or (R,L,C) circuits. It also allows the coupling of different parts in a single circuit in order to have complex geometries like, e.g., spiral coils. It works both in serial and MPP, and is available for the eddy-current solver, soon to be extended to the induced heating and resistive heating ones. A contact capability has also been introduced.
The solver will be introduced, followed by presentations of 2D/3D benchmarks and real life examples.

2 Presentation of the solver

2.1 Presentation of the EM-3D solver

The following is a brief reminder of what was presented in details in [1] and [2].
We will present the "eddy-current" part of the EM solvers since the inductive heating solver consists of iterations of the eddy-current one over a few periods of the source current, and the resistive solver is a simplified version of the eddy-current solver.
In order to solve the Maxwell equations in the "eddy-current" or "diffusion-induction" approximation (where the propagation of the EM waves in the air or vacuum is considered as infinitely fast), we introduce a scalar potential φ and a vector potential $\vec{A}$ such that the magnetic $\vec{B}$ and electric $\vec{E}$ fields can be written respectively as:

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad (1)$$

And

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t} \qquad (2)$$

When choosing an appropriate Gauge condition on $\vec{A}$, the Maxwell equations in the eddy current approximations can then write:

$$\nabla.\sigma\vec{\nabla}\varphi = 0 \qquad (3)$$

And

$$\sigma\frac{\partial \vec{A}}{\partial t} + \vec{\nabla} \times \frac{1}{\mu}\vec{\nabla} \times \vec{A} + \sigma\vec{\nabla}\varphi = \vec{j_s} \qquad (4)$$

Where σ is the electrical conductivity, μ the permeability and $\vec{j}_s$ a source current.
We then use nedelec type basis functions which are particularly well suited for differential geometry and allow precise representations of the exterior derivatives, gradient, curl and divergence.

Equations (3) and (4) projected over the basis functions read:

$$\int_{\Omega} \sigma \vec{\nabla}\varphi . \vec{\nabla} W^0 d\Omega = 0 \qquad (5)$$

And

$$\int_{\Omega} \sigma \frac{\partial \vec{A}}{\partial t} . \vec{W}^1 d\Omega + \int_{\Omega} \frac{1}{\mu} \vec{\nabla} \times \vec{A} . \vec{\nabla} \times \vec{W}^1 d\Omega = - \int_{\Omega} \sigma \vec{\nabla}\varphi . \vec{W}^1 d\Omega + \int_{\Gamma} \frac{1}{\mu} \left[\vec{n} \times \left(\vec{\nabla} \times \vec{A} \right) \right] . \vec{W}^1 d\Gamma \qquad (6)$$

Where W^0 are the so called "0-form" basis functions, scalar functions associated with the nodes of the mesh and $\vec{W}^1$ the 1-form, vector functions associated with the edges.
The last term in Equation (6) is computed using a BEM method. This allows to forget about meshing the air or vacuum surrounding the conductors which is very important when dealing with very small and intricate inter-conductor gaps, as well as moving conductors.

The BEM method requires the introduction of an extra variable $\vec{k}$, called the surface current which does not represent a physical field, but which is such that it creates the same magnetic field and vector potential outside of the conductors as the actual current density flowing through the conductors. Using the BEM method thus also allows to have an extra handle on the current flowing through the conductors and to impose constraints such as having a given total current vs time in a given conductor, or having a current flowing through one conductor equal to the one flowing through another one.

In this FEM-BEM method, $\vec{k}$ and $\vec{A}$ are coupled by the following equations:

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int_{\Gamma} \frac{1}{|\vec{x} - \vec{y}|} \vec{k}\,(\vec{y}) dy \qquad (7)$$

And

$$\left[\vec{n} \times \left(\vec{\nabla} \times \vec{A} \right) \right](\vec{x}) = \frac{\mu_0}{2} \vec{k}(\vec{x}) - \frac{\mu_0}{4\pi} \int_{\Gamma} \frac{1}{|\vec{x} - \vec{y}|^3} \vec{n} \times \left[(\vec{x} - \vec{y}) \times \vec{k}(\vec{y}) \right] dy \qquad (8)$$

Where this last equation allows to get the last term of (6).
In these 2 last equations, we can see the 2 kernels that we have to deal with are:

$$G^0_{3d}(\vec{x}, \vec{x}') = \frac{1}{|\vec{x} - \vec{x}'|} \qquad (9)$$

$$G^1_{3d}(\vec{x}, \vec{x}') = \vec{\nabla} x \frac{1}{|\vec{x} - \vec{x}'|} = - \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'^3|} \qquad (10)$$

We mention these kernels because they will represent the main difference between the 3D and 2D solvers.

2.2 The EM 2D solver

2.2.1 EM fields in 2D axisymmetric - toroidal and poloidal vectors

We now introduce a cylindrical system of coordinates (r,θ,z) and consider that we have some axi-symmetric conditions, i.e. that the fields depend only on r and z. We also suppose that the vector potential is purely azimuthal:

$$\vec{A}(\vec{r}) = A(r, z) \overrightarrow{e_\theta} \qquad (11)$$

Since $\vec{A}$ is homogeneous to $\vec{\nabla}\phi$ (see Equation (2)), we must also have $\vec{\nabla}\phi$ azimuthal and (axisymmetric). We thus have :

$$\phi(r,\theta,z) = \phi(\theta) \tag{12}$$

And

$$\vec{\nabla}\Phi = \frac{1}{r}\frac{\partial\phi}{\delta\theta}\overrightarrow{e_\theta} \tag{13}$$

Were $\frac{\partial\Phi}{\delta\theta}$ cannot depend on θ due to the axisymmetry, so it must be a constant, which value we can relate to the voltage U applied at the ends of a loop in θ by $\frac{\partial\Phi}{\partial\theta} = \frac{U}{2\pi}$, so that $\Phi(0) = 0$ and $\Phi(2\pi) = U$.
We thus have:

$$\vec{\nabla}\Phi = \frac{U}{2\pi r}\overrightarrow{e_\theta} \tag{14}$$

The other fields read:

$$\vec{B}(\vec{r}) = B_r(r,z)\overrightarrow{e_r} + B_z(r,z)\overrightarrow{e_z} \tag{15}$$

$$\vec{E}(\vec{r}) = E_\theta(r,z)\overrightarrow{e_\theta} \tag{16}$$

$$\vec{j}(\vec{r}) = j_\theta(r,z)\overrightarrow{e_\theta} \tag{17}$$

The vectors parallel to $\overrightarrow{e_\theta}$ are called toroidal or azimuthal, whereas these lying in the $(\overrightarrow{e_r}, \overrightarrow{e_z}\}$ plane are called poloidal.

2.2.2 FEM in 2D

The EM 2D mesh is defined by a set of faces (the elements) with their nodal connectivities. The azimuthal vectors are represented by 0-form basis functions w^0 which are associated with the nodes of the mesh. The poloidal vectors are represented by 1-form basis functions $\vec{W}^1$ associated with the edges of the mesh. The computation of the 2D FEM matrices is very similar to the ones of the 3D FEM matrices, where the integration now takes place over faces instead of 3D elements.

2.2.3 BEM in 2D

The BEM matrices are very different though, due to an explicit iteration over θ which changes the kernels.
For example, if we take Equation (7), write it in cylindrical coordinates and take into account the axisymmetry :

$$\vec{A}(r,z) = \iiint r'dr'dz'd\theta' \frac{k(r',z')\overrightarrow{e_{\theta'}}}{|x(r,\theta,z) - x'(r',\theta',z')|} \tag{18}$$

We have:

$$A(r,z) = \vec{A}(r,z).\overrightarrow{e_\theta} = \iiint r'dr'dz'd\theta' \frac{k(r',z')\overrightarrow{e_{\theta'}}.\overrightarrow{e_\theta}}{|x(r,\theta,z) - x'(r',\theta',z')|} = \iint dr'dz'k(r',z')G(r,z;r',z') \quad (19)$$

Which gives, after a few manipulations [3] [4], the 2D kernel:

$$G(r,z;r',z') = \int_0^{2\pi} \frac{\overrightarrow{e_{\theta'}}.\overrightarrow{e_\theta}}{|x(r,\theta,z) - x'(r',\theta',z')|} = \frac{2k}{rr'}\int_0^{\pi/2} \frac{2\sin^2\theta - 1}{\sqrt{1-k^2\sin^2\theta}}d\theta = \frac{4k}{\sqrt{rr'}}\left[\frac{K(k)-E(k)}{k^2} - \frac{K(k)}{2}\right] \quad (20)$$

Where we introduced the elliptic integrals of the first kind

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} \quad (22)$$

And of the second kind:

$$E(k) = \int_0^{\pi/2} \sqrt{1-k^2\sin^2\theta}\, d\theta \quad (23)$$

And in Equation (20) :

$$k^2 = \frac{4rr'}{(r'+r)^2 + (z'-z)^2} \quad (24)$$

In summary, the big difference in the BEM method between 2D and 3D is the new 2D kernel:

$$G_{2d}^0(\vec{x},\vec{x}') = G_{2d}^0(r,z;r',z') = \frac{4k}{\sqrt{rr'}}\left[\frac{K(k)-E(k)}{k^2} - \frac{K(k)}{2}\right] \quad (25)$$

Which compares to Equation (9) in 3D. The gradient of the kernel (10) is computed using the same method.

3 Features of the 2D axisymmetric solver

3.1 Coupling of EM-2D with mechanical and thermal 3D

Since LS-DYNA is primarily a 3D code, where most of the features are available only in 3D, we decided to couple the EM-2D with the 3D LS-DYNA. This means that the user needs to provide a 3D mesh as well as, for each conducting part, a segment set to define the plane where the EM-2D is done. Once the EM fields are computed in 2D on this plane, they are just reported over the full 3D mesh by rotations around the axis, since the EM fields are supposed to be axisymmetric. Solving the EM in 2D on a plane and applying rotations to get the 3D fields is much faster than computing the EM fields in 3D. The user needs to make sure that the mesh of the conductor parts actually have the axisymmetric invariance. However, the simulation does not need to be made on a full 360^0, but can be made on a slice of the full cylinder. This slice needs to be $(\frac{1}{2})^p$ times the full cylinder ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...), if applying the proper mechanical and thermal boundary conditions at the 2 ends of the slice (no out of plane motion). Figure (1) shows a set up of such a simulation.

***EM_2DAXI**

PID	SSID			StartSSID	EndSSID	NumSec	
3	3			1	2	32	

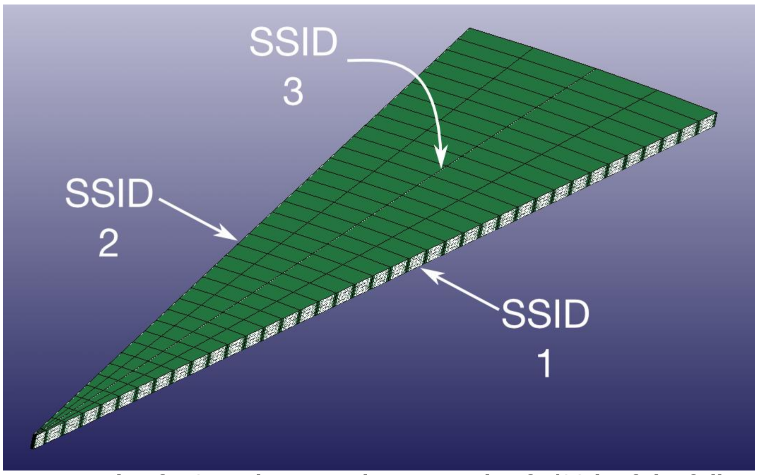

Figure 1 Example of a 2D axisymmetric part made of 1/32th of the full cylinder

The Lorentz force and Joule heating are also computed in 2D and then reported by rotations over the 3D mesh so that the 2D EM can be coupled with 3D mechanics and thermal.

3.2 Coupling the EM 2D with external circuits

All the ***EM_CIRCUIT** types existing in 3D can be used in 2D.
For imposed current circuits, the total current through the part is imposed in the BEM system, since part of the variables represent the total currents flowing through each different circuits.
For imposed voltage circuits, including R,L,C circuits, each voltage drop V is imposed on the scalar potential φ using Equation (14).

3.3 Connecting different circuits together

As with the 3D solver, one can connect different circuits together, i.e. impose a linear constraint on the global currents of 2 circuits $(c_1 i_1 + c_2 i_2 = 0)$, like, for example $i_1 = i_2$, using the ***EM_CIRCUIT_CONNECT** card. This allows for example the simulation in 2D of a spiral or helix coil. This circuit connect feature can even be used when the coil is connected to a R,L,C circuit, where the best results are when the R,L and C of the coil are evenly spread between the different 2D circuits. See Figure (2) ($R_i = \frac{R}{N}, L_i = \frac{L}{N}, Vo_i = \frac{Vo}{N}, C_i = NC$ if N is the number of loops).

***EM_CIRCUIT_CONNECT**

CONNID	CONNTYPE	CIRCID1	CIRCID2	C1	C2		
1	1	1	2	1	-1		

CONNID	CONNTYPE	CIRCID1	CIRCID2	C1	C2		
2	1	2	3	1	-1		

CONNID	CONNTYPE	CIRCID1	CIRCID2	C1	C2		
3	1	3	4	1	-1		

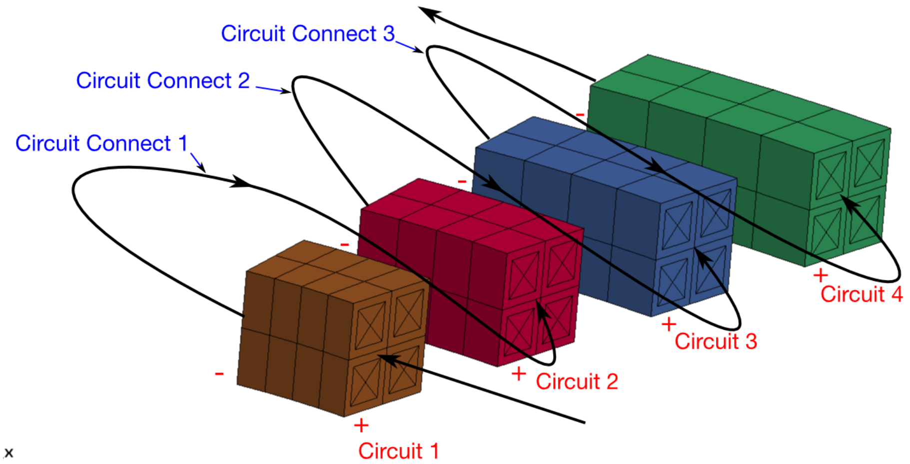

Figure 2 Example of a spiral coil modelling using the axi symmetric solver and the circuit connect card

3.4 EM contact in 2D

The EM contact is available in the 2D solver. The contact uses the same cards as the 3D solver, and basically works the same way, with a local re-building of the BEM mesh around the contact areas.

3.5 EM-2D coupled with EM-3D

A coupling between the EM-2D and EM-3D is currently under development. This coupling will allow to model parts with nearly axisymmetric invariance, like a circular, spiral, or helical coil, along with other parts which do not have this symmetry. This should make the resulting simulations much faster than full 3D ones.

4 Numerical results

4.1 Helicoidal coil connected to a RLC circuit

The first example is a ring expansion using a helicoidal coil. A R,L,C circuit will be used in order to highlight the capabilities of the *EM_CIRCUIT_CONNECT card. Figure (3) shows the 2D set up and the 3D equivalent set up. Figure (4) shows the current density at a given time during the run and Figure (5) offers a comparison of the maximum displacement between the 2D and the 3D run. Agreement is quite good, and the slight disprepancies may be explained through some small 3D effects. The 3D problem took 2 hours on 24 CPUs while the 2D problem took 15 minutes on 1 CPU resulting in a significant gain of time.

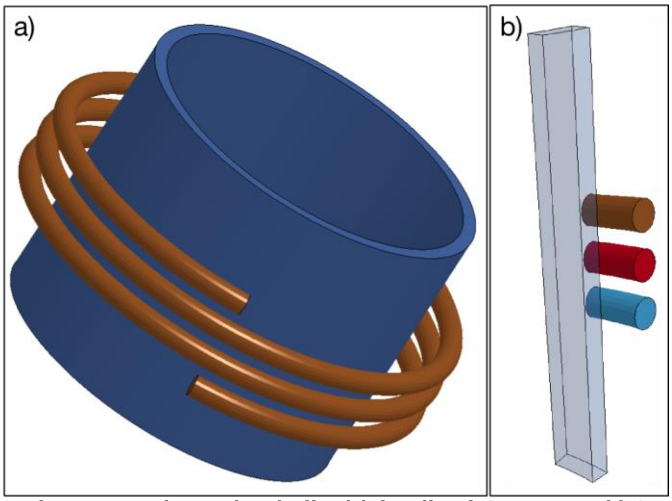

Figure 3 Ring expansion using helicoidal coil : a) 3D set up, b) 2D set up.

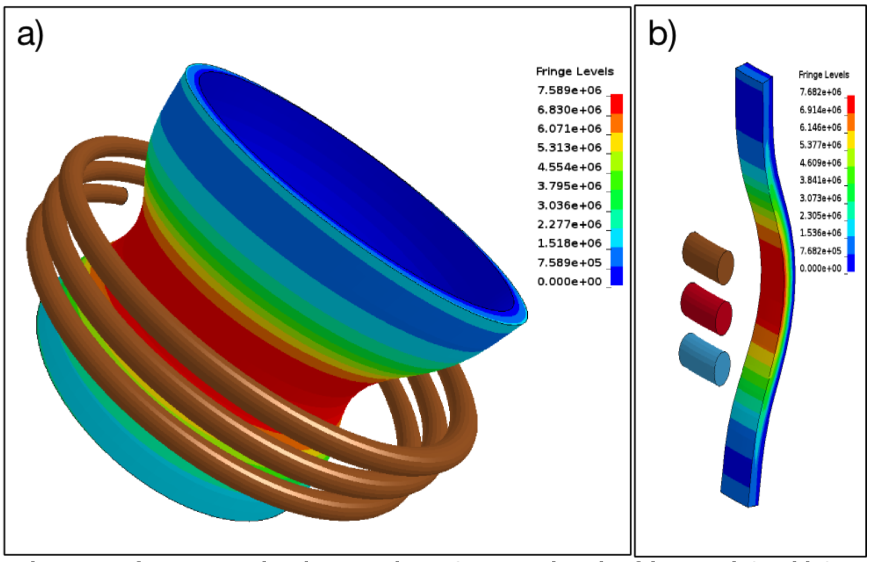

Figure 4 After expansion is complete. Current density fringes a) 3D, b) 2D

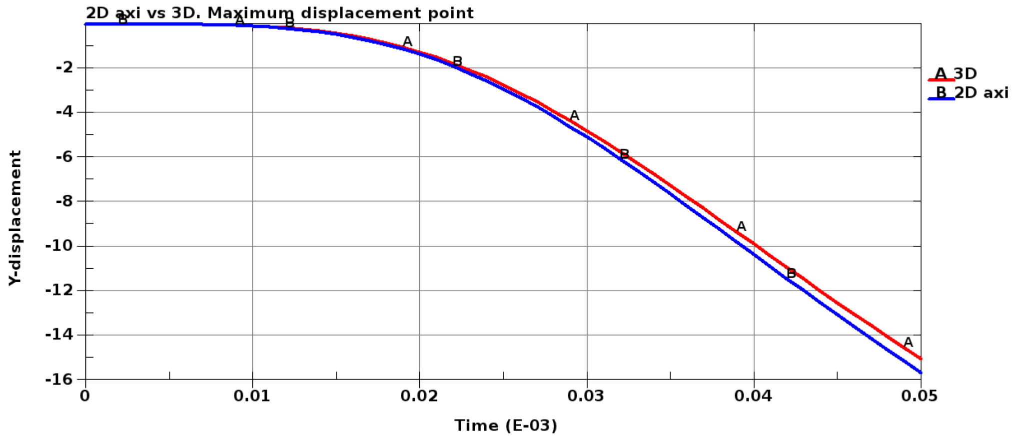

Figure 5 Maximum displacement point. Comparison between 3D (in red) and 2D axisymmetric (in blue).

4.2 Magnetic forming with spiral coil

This case is a 2D version of the classic metal forming example available as a tutorial example on the LSTC website and presented in more detail in [1]. Figure (6) shows the 3D and 2D setups. The same die mesh can be used in both cases since it is an insulator and will be ignored by the EM solver. Figure (7) shows the current at different times between the 2D and 3D runs while Figure (8) shows the displacement at different locations. Again, agreement between the results are good while discrepencies can be explained by 3D effects due to the shape of the coil. Indeed, the pitch of the spiral is not taken into account in 2D, where the coil is like a set of tori. This time, the 3D run took 5 hours and 30 minutes on 12 CPUs while the 2D run took 5 minutes on 1 CPU.

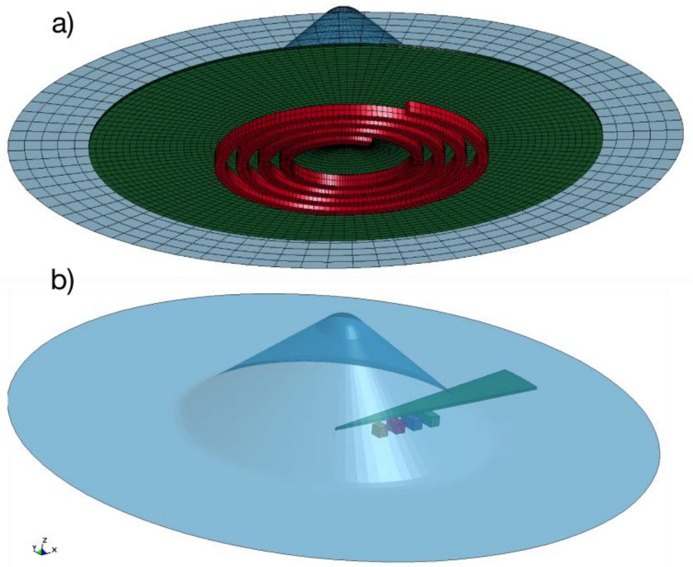

Figure 6 Magnetic metal forming with spiral coil : a) 3D setup, b) 2D set up. In both cases, the same die (in blue) is used.

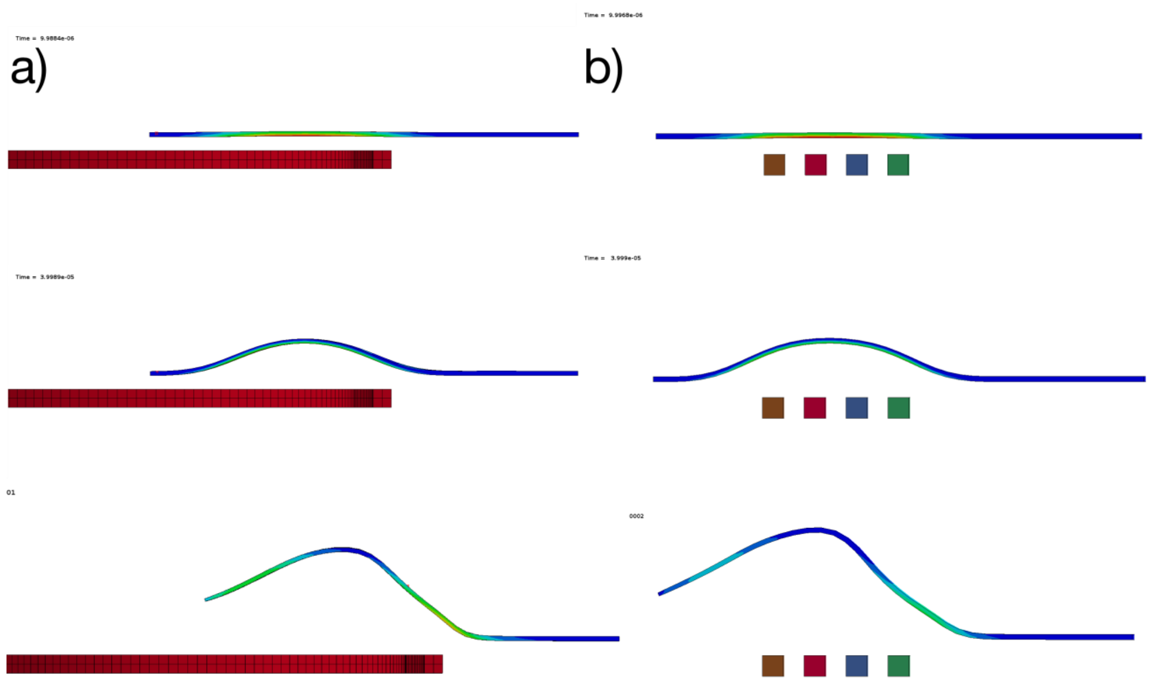

Figure 7 Displacement of the workpiece at different times. Comparison between 3D, a) and 2D b).

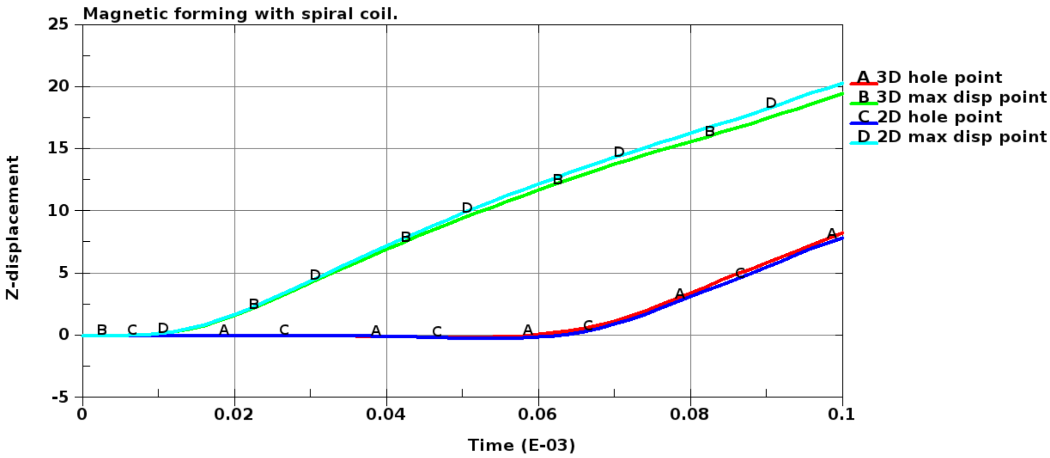

Figure 8 Comparison of displacements for a point near the center hole and the point of maximum displacement.

5 Conclusion

The new axisymmetric solver for EM was introduced. Among its features are MPP capabilities, the introduction of the circuit connect card which allows the modelling of helicoidal and spiral shaped coils as well as the use of R,L,C circuits and the possibility to model contact between conductors. Future developments will include the mixing of 2D and 3D parts in order to offer further options and tools to users who wish to reduce their calculation time and in order to run multiple trials for optimization purposes and potential coupling with LS-OPT.

6 References

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