Recent Advances on Surrogate Modeling for Robustness Assessment of Structures with respect to Crashworthiness Requirements

Fabian Duddeck[1], Erich Wehrle[1]

[1]Technische Universität München, Chair of Computational Mechanics
Arcisstr. 21, D-80333 Munich, Germany, Email: duddeck@tum.de

1 Introduction

Due to the inherent nonlinearity, crashworthiness is one of the most demanding design cases for vehicle structures. Recent developments have enabled very accurate numerical simulations and corresponding optimizations. Therefore, structural concepts are now much better adapted to the specific requirements. This has led to designs in which redundancies are reduced and highly effective car concepts have been derived. Important questions are then the reliability and the robustness of designs. Reliability addresses probabilities that constraints are violated and robustness assures that performance loss is small due to unavoidable variations.

Because there is neither consensus on the precise definitions of robustness and reliability in the field of crashworthiness nor is there a unique understanding of the appropriate numerical methods, this paper tries to clarify these aspects. Further an overview of recent research results from PhD theses supervised by the first author is presented. Hereby, special focus is laid on physical surrogate models for robustness assessments.

Physical surrogates enable robustness investigations in early design phases considering mainly uncertainties related to lack-of-knowledge (changes, which occur later in the development process). Solution spaces derived by simplified models, here surrogate models based on lumped mass approaches, allow decoupled development of components. Robustness is achieved here by maximizing these solution spaces, resulting in high design flexibility. Further, an approach for robust design optimization is presented for later development phases based on a trust region and multi-fidelity approach. Low-fidelity models (physical surrogates using equivalent static loads or sub-structures) are used in the explorative phase of the analysis. Manufacturing or load case uncertainties are considered. Special criteria are established to switch to high-fidelity models (nonlinear transient finite element models) whenever necessary during optimization. It is hence possible to include robustness into the optimization with reduced numerical effort.

KEYWORDS Crashworthiness, Uncertainties, Robustness, Lack-of-knowledge, Physical Surrogates, Multi-fidelity Approach, Optimization.

2 Surrogate Modeling for Crashworthiness

Today, crashworthiness is mainly assessed virtually by Finite Element Methods (FEM). Due to high model complexity, the computations require several hours even if they are run on a higher number of CPUs and with advanced parallel computing; hereby, it is normally accepted that a single assessment is finished overnight. The computational effort for these *high-fidelity models* is now one of the main difficulties to be addressed by optimization methods, e.g. [1, 2]. The same is true for uncertainty assessments where the structural performances are analyzed with respect to unavoidable fluctuations in design and noise parameters as described in Section 3. For both, optimization and robustness, we need normally at least 200 or 300 simulations to obtain rough estimates and up to several thousand computations for more accurate studies. This is clearly not feasible on a daily basis. To overcome this difficulty, so-called *surrogate models* are often proposed. Here, the high effort computation is replaced by simpler and faster approaches. Most of the methods in the literature are based on *mathematical surrogates* where an initial sampling (normally by design of experiment techniques, DoE) is used to construct a mathematical approximation of the response(s), see Section 2.1. Alternatively, simplified models (here called *physical surrogates*), which still include some physical characteristics, can be used, see Section 2.2. The paper at-hand focuses on these physical surrogates or *low-fidelity models*.

2.1 Mathematical Surrogates

Most of the current optimization software tools, e.g. LS-OPT[1], optiSLang[2], or ClearVu Analytics[3], offer a wide range of methods for mathematical surrogates. These are also known as meta-models and regression models and are used in Response Surface Methods (RSM) to reduce computational effort of the design optimization process. Some of the most used methods are generalized linear models, [3], decision trees or random forests, [4], support vector machines [5], Gaussian processes (also known as Kriging), [6], and moving least squares, [7]. Further approaches are based on fuzzy models, artificial neural networks, or radial basis functions (see [3] for an overview and [8] for a discussion). Further results can be expected from the recently started German research project eEgO[4].

All these approaches have in common that they use high-fidelity FEM simulations to generate the responses for a certain number of samples, which is ideally done in an adaptive manner. The approximation of these responses between (or more questionable outside) these sample points via the different mathematical approaches mentioned above are then used for optimization as well as uncertainty assessment. The usability of these techniques depends on the quality of the approximations, which can be quantified by for example leave-one-out approaches for interpolation methods or by measures like the coefficient of determination for regression methods. It is essential that the error made in the response approximations is considered in the assessments and optimizations. Unfortunately, this is not always realized, e.g. [9]; if a surrogate model is used to assess robustness or reliability (see Section 3 for the difference), the error of the response approximation has to be included into the estimates of failure probabilities (reliability) or performance losses (robustness) if these are evaluated via the surrogates. Hereby it is often not sufficient to consider global error measures; nevertheless obtaining appropriate local error estimates remains challenging. This is especially true for crashworthiness because smoothness of the responses cannot always be assumed and bifurcations due to stability or near contact situations lead to high local incorrectness of the meta-models. These drawbacks motivate to investigate physical surrogates for robustness as discussed in the next section.

2.2 Physical Surrogates

Physical surrogates are simplified computational models, which still model physical performances via stress analysis, material models, contact etc. The existing approaches can be classified as follows:

- **Sub-structure modeling approach:** by cutting out a part of the total vehicle structure, either as some kind of box or along the component interfaces, e.g. [10]. The sub-structure is then analyzed considering special interface conditions, e.g. guided deformation histories over time. The responses are normally relatively correct as long as the sub-structure is only changed slightly.
- **Hybrid nonlinear FE–rigid body approach:** where a part of the total model is replaced by rigid bodies, e.g. [11]. Here the influence of the parts which are considered rigid is neglected, which is often questionable especially in high speed impacts.
- **Hybrid nonlinear FE–elastic FE approach:** An alternative is a hybrid modeling where a part is considered to be only elastic and eventually computed via an implicit FE solver. Because the coupled explicit-implicit FE methods are still in their infancy, the potential of this approach is not fully exploited.
- **Hybrid fine–rough FE mesh**: Another hybrid approach may consider FE models with a different degree of mesh refinement. The practicality of this approach is not always given because of the complexity of current meshing procedures. An automated modification of a FE mesh may be only feasible for components and not for full vehicle structures.
- **Space mapping techniques** have been proposed to establish the simultaneous usage of the fine and coarse models, e.g. [12, 13]. To the opinion of the authors, this version like the other possibilities of multi-fidelity approaches is promising.
- **Multi-body system approaches** have also been investigated, e.g. [14]. They belong to the group of simplified models where masses are lumped and connected via eventually nonlinear spring and damper systems, e.g. [15].
- **Analytic and semi-analytic crash modeling**: In [16], a part of the FE model was replaced by semi-analytic crash models based on the theory of collapse and limit analysis, which is an interesting approach worth further studies.

1 http://www.dynamore.de/de/produkte/opt/ls-opt
2 http://www.dynardo.de/software/optislang.html
3 http://www.divis-gmbh.de/en/clearvu-analytics.html
4 https://www.asc-s.de/de/news/2/bmbf-forderung-fur-projekt-eego/

- **Equivalent static load methods:** Finally, equivalent static load methods have been proposed where linear elastic computations are used instead of a nonlinear transient analysis, [10, 17].

In contrast to the mathematical surrogates, the approaches listed here still include physical behavior, although it is not sufficiently well investigated if all relevant physical characteristics are covered. For example, a contact situation might change if the design is modified in an optimization or uncertainty analysis. If only a sub-structure is considered but the new contact situation is happening between it and the rest-structure, this may be not regarded via surrogate modeling. An approach proposed in [18] may improve here and in general the usage of surrogates and especially physical surrogates for crashworthiness. Nevertheless, it is here very important to reflect carefully the required accuracy of the surrogate modeling. In general, methods for early development phases should be quite different from those employed in the later design stages. This relates directly also to the type of uncertainties to be considered. In the early development phase, uncertainties are mainly due to lack-of-knowledge while later close to the SOP (start of production), they originate from variability due to manufacturing processes and load conditions. Together with a clarification of the difference between reliability and robustness, this is addressed in the following section.

3 Robustness

In the literature, a distinction is made between *epistemic* and *aleatoric uncertainties, e.g.* [19, 20]. The former relates to aspects which could be known in principle but are not known practically. For example, uncertainties introduced in the derivation of the model and the corresponding simplifications can be considered to be epistemic. Aleatoric uncertainty refers to physical variability and randomness present in the system being analyzed or in its environment (e.g. variation of thickness, shape, material due to manufacturing or fluctuations in load conditions like impact angle or barrier position). Here, a probabilistic modeling using stochastic distributions (mean value, standard deviation, etc.) can be employed in contrast to a possibilistic approach via fuzzy theory or interval analysis [21] for the epistemic case.

In this paper, two aspects of robustness are addressed. First, a special type of epistemic uncertainty[5] is regarded to assess robustness in early development phases. The corresponding *lack-of-knowledge* uncertainty results from decisions made later in the development process. Because these aspects cannot be known in principle at the time of investigation, it is reasonable to distinguish this from the definition given above for epistemic uncertainty; it is an irreducible uncertainty. Additionally, it is not so strongly related to risk analysis, i.e., we do not want to reduce variability but we want to enlarge the intervals to obtain high flexibility. Here, a robust decision is a decision, which can be adapted easily to constraints or modifications known only later in the development process. As a second robustness case, aleatoric uncertainty is considered in the later development phases.

The above mentioned uncertainties can be used in *reliability* or in *robustness* studies; often both investigations are combined especially when they are used within an optimization. The distinction between robustness and reliability is not always made in full clarity; hence a brief definition is given here. Reliability is concerned with the *probability of constraint violation* while robustness regards *loss in performance criteria*. In an aleatoric context, the former shifts the stochastic cloud away from the constraint limits while the latter reduces variance of the performances. For lack-of-knowledge uncertainty, this is normally different; here *safety margins* may be introduced for reliability and *flexibility* is proposed here as measure of robustness. This means that in the early development phase, designs are robust, which have a high flexibility with respect to decisions made later in the process.

3.1 Physical Surrogates for Robustness for Lack-of-knowledge Situations

In the early development phase, decisions have to be made fast without the availability of detailed models. Here, a recently proposed approach via *solution spaces* is attractive (see [15, 22]), where an idea based on system engineering (V-model as visualized in Fig. 1) is realized to decouple the components and different functions of a car body with respect to crashworthiness enabling independent component development. Solution spaces are then defined by introducing constraints related a special crash case (e.g. for a frontal high speed impact with a full width rigid wall the

[5] A discussion of epistemic uncertainty is given for example in S. Ferson, C.A. Joslyn, J.C. Helton, W.L. Oberkampf, and K. Sentz: "Summary from the epistemic uncertainty workshop: consensus amid diversity", Reliability Engineering and System Safety **85** (2004) 355–369.

constraints of crash pulse, intrusion and order of deformation are used [15]). Inside of the feasible design area described by the limit functions of the constraints a hypercube is searched to avoid coupling of the different components; hence independent upper and lower limits, i.e. a corridor, can be found for the force-displacement curves of each component. They can then be optimized independently such that their performance is lying within the corridor. Full vehicle analyses are only required for final validations; the computational effort for optimization is reduced strongly, see Fig. 2. Reliability (without assessing probabilities) may be improved by safety margins on the corridors. This is not considered further because this paper is focused on robustness, which is related to the optimization objective of high flexibility to enable that developers can react to changes, which occur later in the development (lack-of-knowledge).

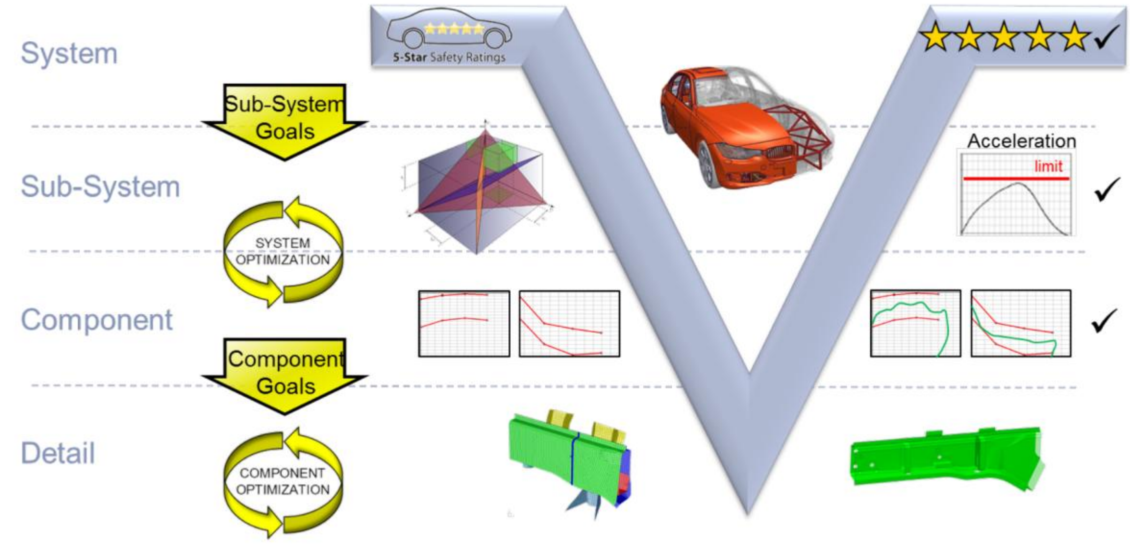

Fig.1: V-model approach to decouple components, [15].

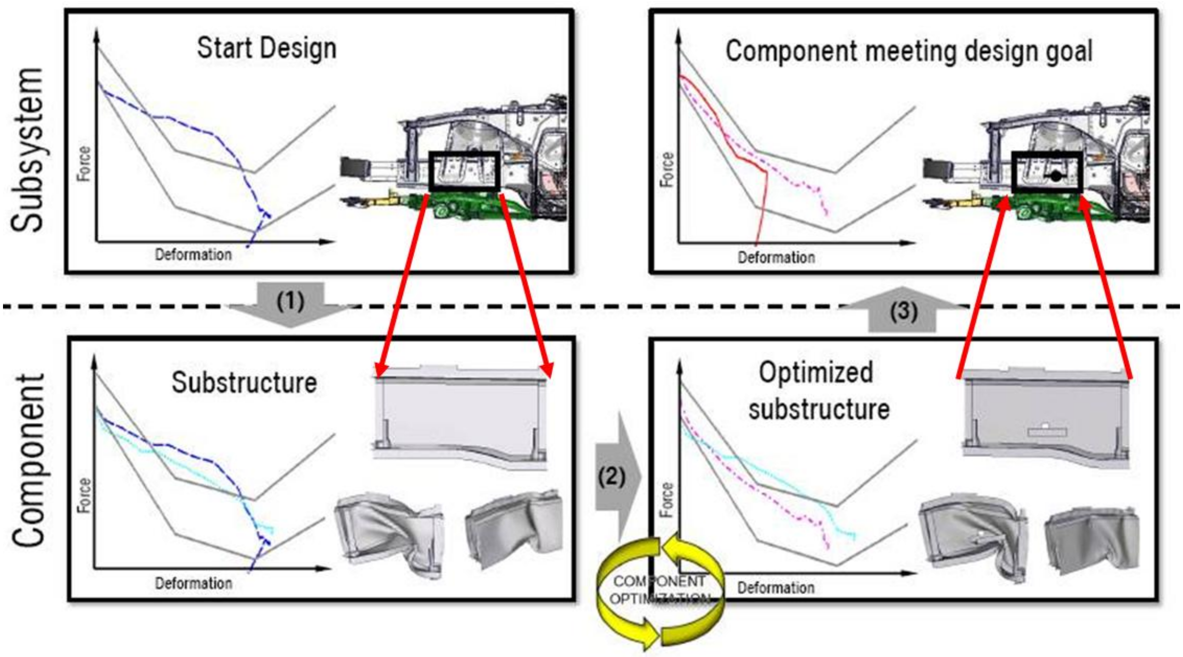

Fig.2: Solution space-based optimization showing a corridor and the initial force-displacement curve from the sub-system simulation (upper left image) and the component simulation (lower left image). Additionally, the result of a component optimization is shown (lower right image) and the performance obtained from validation on sub-system level (upper right image), [15].

The corridors are obtained for each component via physical surrogates. As shown for the example mentioned above, the FE model is replaced by a lumped mass model (Fig. 3) representing the four load paths, which are activated successively. Each load path is sub-divided into sections and for each section a corridor is computed based on the physical surrogate. The result is shown in Fig. 4. This physical surrogate based on lumped masses helps to compute fast the responses and to modify the design with respect to the design requirements. The goal is to achieve high design flexibility for those components, which have high uncertainty in the early design stage with respect to changes later in the development process. The robustness is improved here twofold: first an optimization algorithm modifies the corridors as much as possible to achieve smooth transitions and – more important here – broad corridors. In addition, the component optimization (Fig. 2) modifies shape, thickness etc. of a component such that its force-displacement curve is in the middle of the corridor. Both aspects assure robustness with respect to lack-of-knowledge in the early development phase.

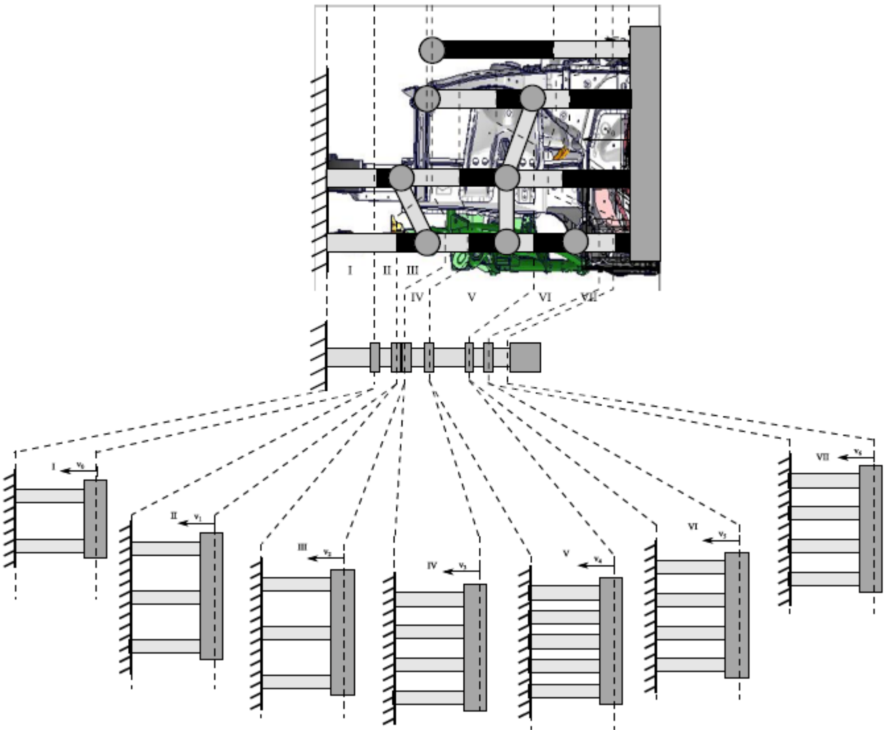

Fig.3: Physical surrogate (lumped mass model) showing the sections of the four load paths, [15].

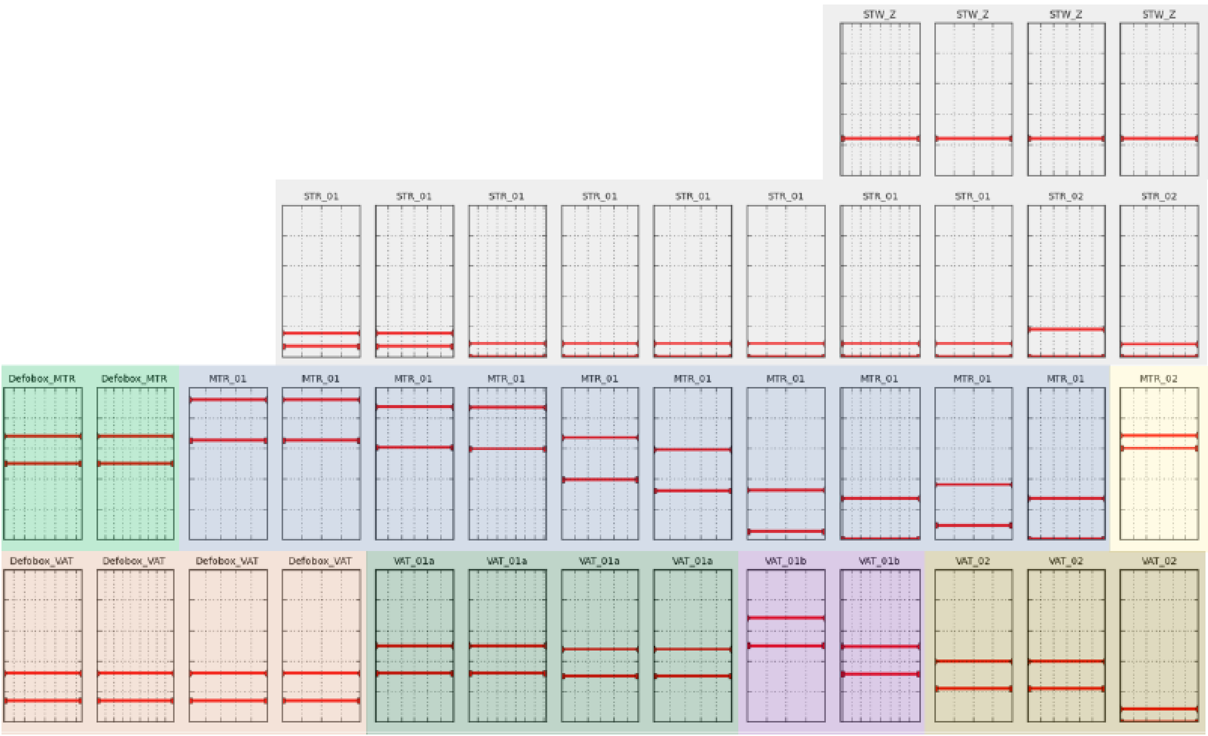

Fig.4: Set of corridors for the four load paths example.

3.2 Physical Surrogates for Robustness at the End of the Development Process

Concerning robustness at the end of the development process, mainly aleatoric uncertainties have to be considered and, therefore, lack-of-knowledge is no longer the main focus. Hence a traditional probabilistic approach can be used. Robustness can be defined here as low sensitivity of the structural responses to unavoidable variations due to manufacturing (e.g. [23] for shape sensitivity studies) and in loading. In an optimization framework, we normally have a multi-criteria optimization problem where additionally to the objectives the standard deviation of the responses is minimized. This means that during the optimization the variance has to be computed, which increases strongly the number of evaluations necessary. Hence surrogate modeling techniques are often used; most of the studies in literature, e.g. [9], employ mathematical surrogates. Compared to this, one of the advantages of physical surrogates lies in the fact that physical characteristics are still embedded into the surrogates. For example, a local buckling or contact situation leading to bifurcations can still be represented by a sub-structure. In addition, it is easier to consider a high number of stochastic design and noise variables in a sub-structure or in equivalent static load computations than in studies based on mathematical surrogates. Therefore, the first studies on physical surrogate-based robust design optimization (RDO) were realized in the frame of a recently finished PhD thesis [10]. The main idea is to perform a robustness analysis on the physical surrogate during the optimization until a critical area (here target interval, TI) in the design space is reached or until the optimization process reaches termination. Then a more accurate robustness assessment is done on the complete non-linear dynamic model. The approach is visualized in Fig. 5; it can be considered as a special type of multi-fidelity approach for crash RDO.

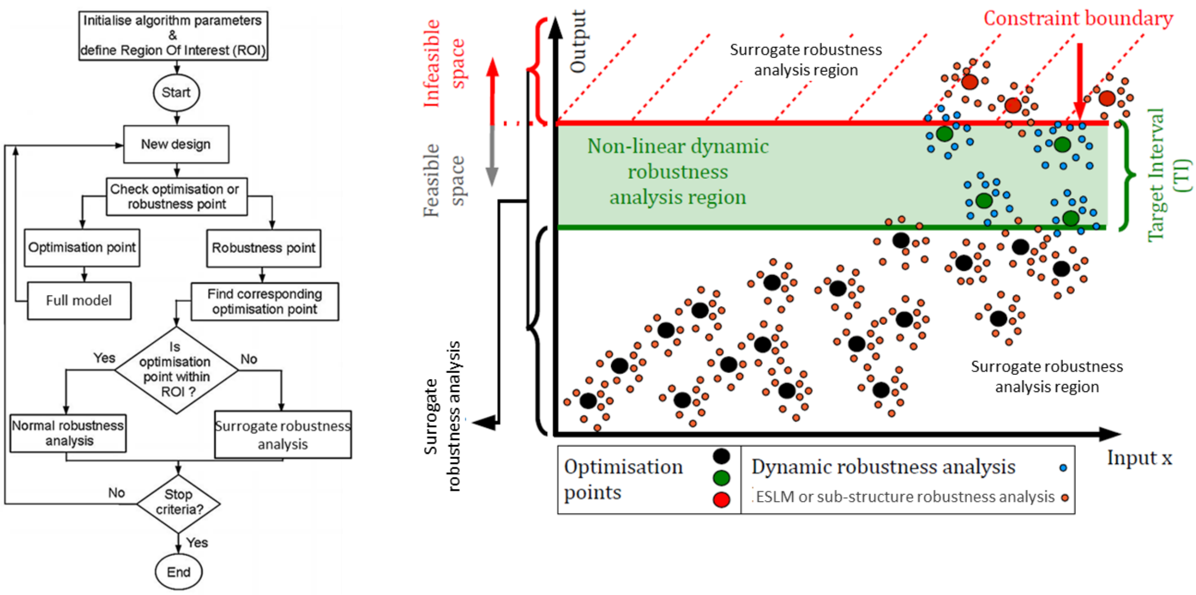

Fig.5: Work flow for the multi-fidelity approach for RDO using physical surrogates (left), and illustration of the target interval (TI) concept as a special region of interest (ROI) formulation (right), modified image taken from [10].

As an example for the sub-structure approach, a side impact model is shown in the left part of Fig. 6 together with the B-pillar as the sub-structure (Fig. 6, right). The generation of this sub-structure requires that the interface conditions are updated automatically when the design is modified remarkably during the optimization; cf. the flow shown in Fig. 7. Appropriate update criteria for this are still under investigation (see the eEgO project mentioned above).

As an alternative physical surrogate, the equivalent static load approach, see [17] for the theory, was investigated. Here, the displacements taken from a non-linear crash analysis are multiplied with the linear elastic stiffness matrix of the vehicle such that equivalent nodal forces are obtained. Then the robustness analysis can be performed via an implicit, linear elastic FEM. This approach is promising and partially easier than the sub-structure method. Nevertheless, in some cases it is difficult to obtain the correct linear stiffness matrix and, even more relevant, some response quantities are difficult to compute via linear elastic simulations (e.g. head impact criterion, HIC). Here more research is needed.

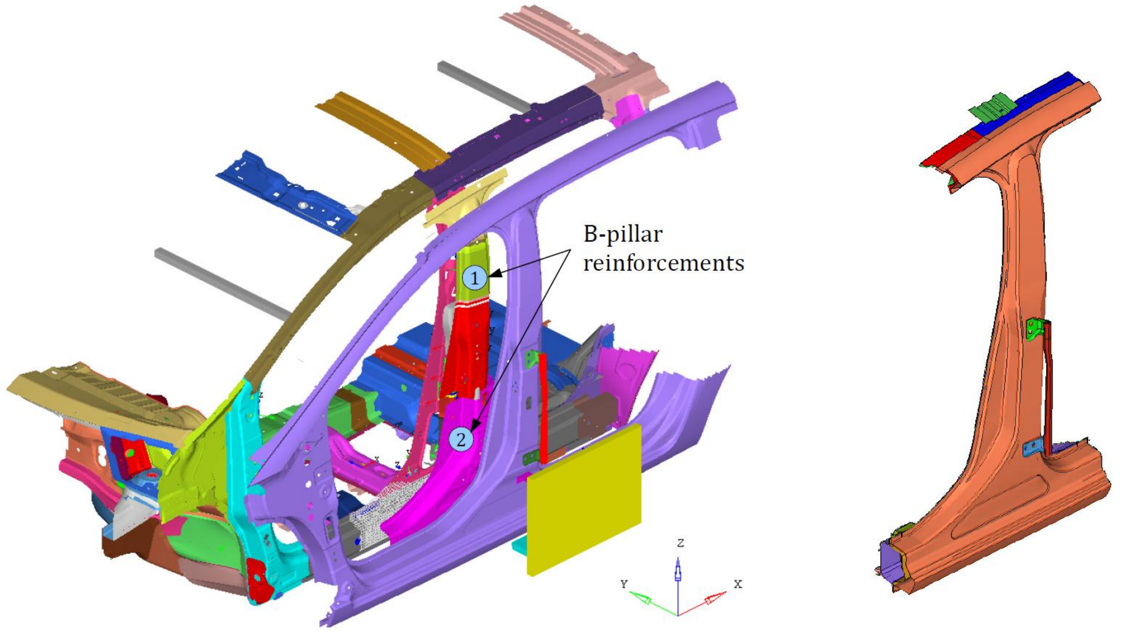

Fig.6: Exploded view of the side impact model (left). The model is already reduced here, i.e. the vehicle is cut at the tunnel, the doors are missing etc. The barrier is modified such that the correct impact energy is considered; sub-structure for robust design optimization (right) [10].

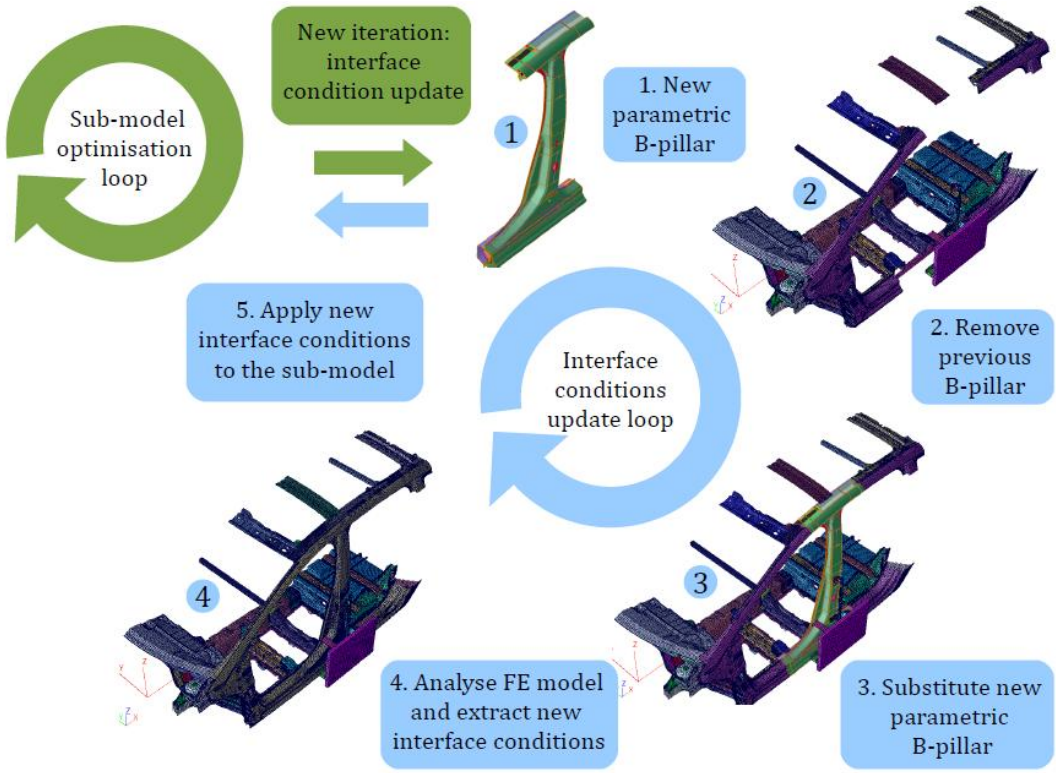

Fig.7: Work flow for the update of the interface conditions where the sub-structure (B-pillar) is re-embedded into the total model. In the sub-model optimization loop (top left part of the image), the robustness is assessed on the physical surrogate, i.e. via the sub-structure, [10].

The principle of equivalent static loads for robustness assessments is visualized in the flow diagram of Fig. 8. Finally, in Fig. 11, the flow diagram for the validation case considered in the next section is shown where a combined approach (equivalent static loads and sub-structure) is used.

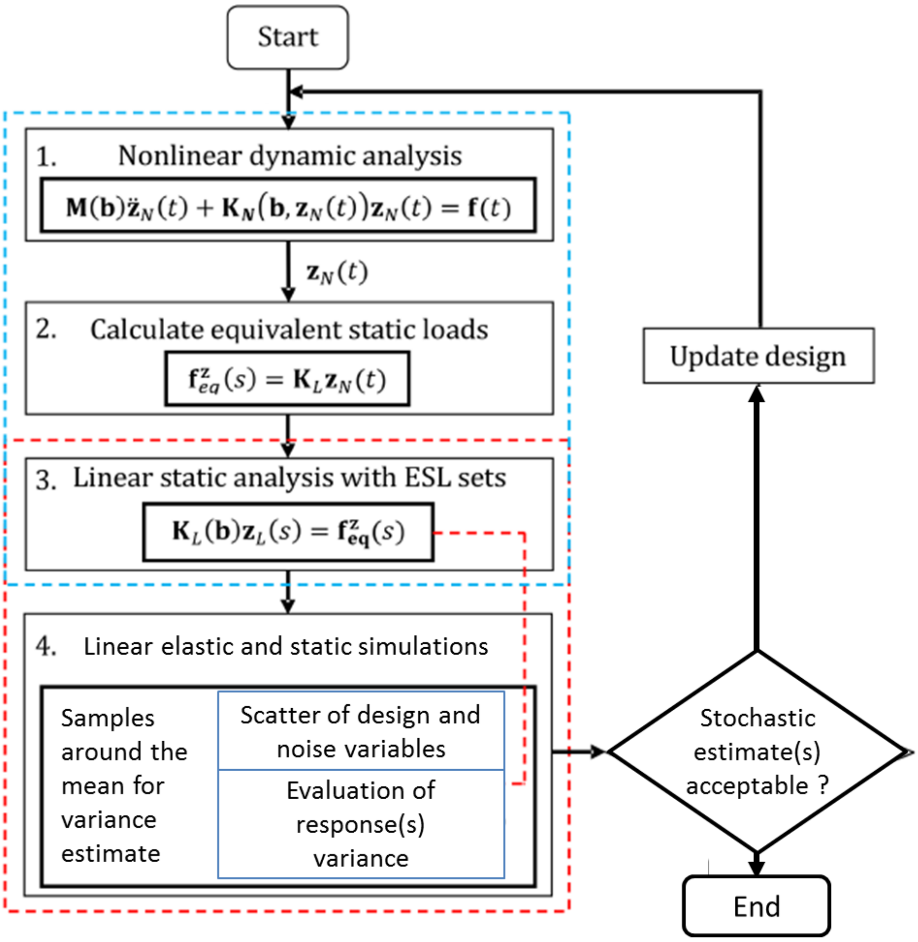

Fig.8: Flow diagram for the physical surrogate approach based on equivalent static loads, after [10].

4 Example

As illustration of the approach, a rocker under pole impact is regarded as shown in Fig. 9. A smaller part of the rocker is used as sub-structure and the interior reinforcements of the rocker are used as design variables (shape parameters via a modeling by SFE CONCEPT[6]).

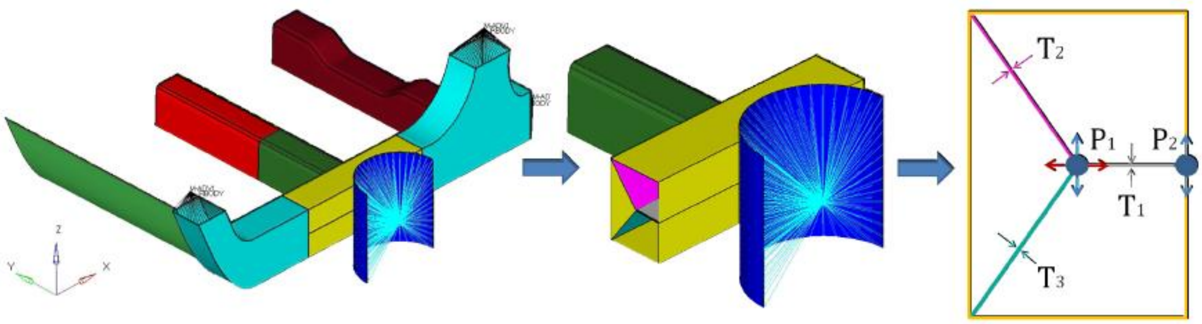

Fig.9: Illustration example for combined RDO approach using a sub-structure (middle) and equivalent static loads. The right part of the image shows the design variables, [10].

Table 1 gives the data for this example. Noise variables are mass and velocity of the impactor and design variables are three thicknesses and two shape parameters.

6 http://www.homepage.sfe-group.org/produkte/sfe-concept/

	Variable	Lower limit	Initial design	Upper limit	Distribution	Standard deviation
Design variables (mm)	P_z	-50	0	50	normal	1.0
	P_y	-40	0	40	normal	1.0
	t_1	0.5	1.0	1.5	normal	0.07
	t_2	0.5	1.0	1.5	normal	0.07
	t_3	0.5	1.0	1.5	normal	0.07
	Variable	Lower limit	Initial design	Upper limit	Distribution	Standard deviation
Noise variables	M_0 (kg)	98.5	100	101.5	uniform	NA
	V_0 (m.s^{-1})	7.91	8.06	8.21	uniform	NA

Table 1: Overview of the design and noise variables considered in the validation case, [10].

The objective of the RDO is to reduce the mass of the rocker while respecting an intrusion constraint. A 2σ approach is used for the constraint for robustness analysis of designs. The progress of the optimization is shown in Fig. 10.

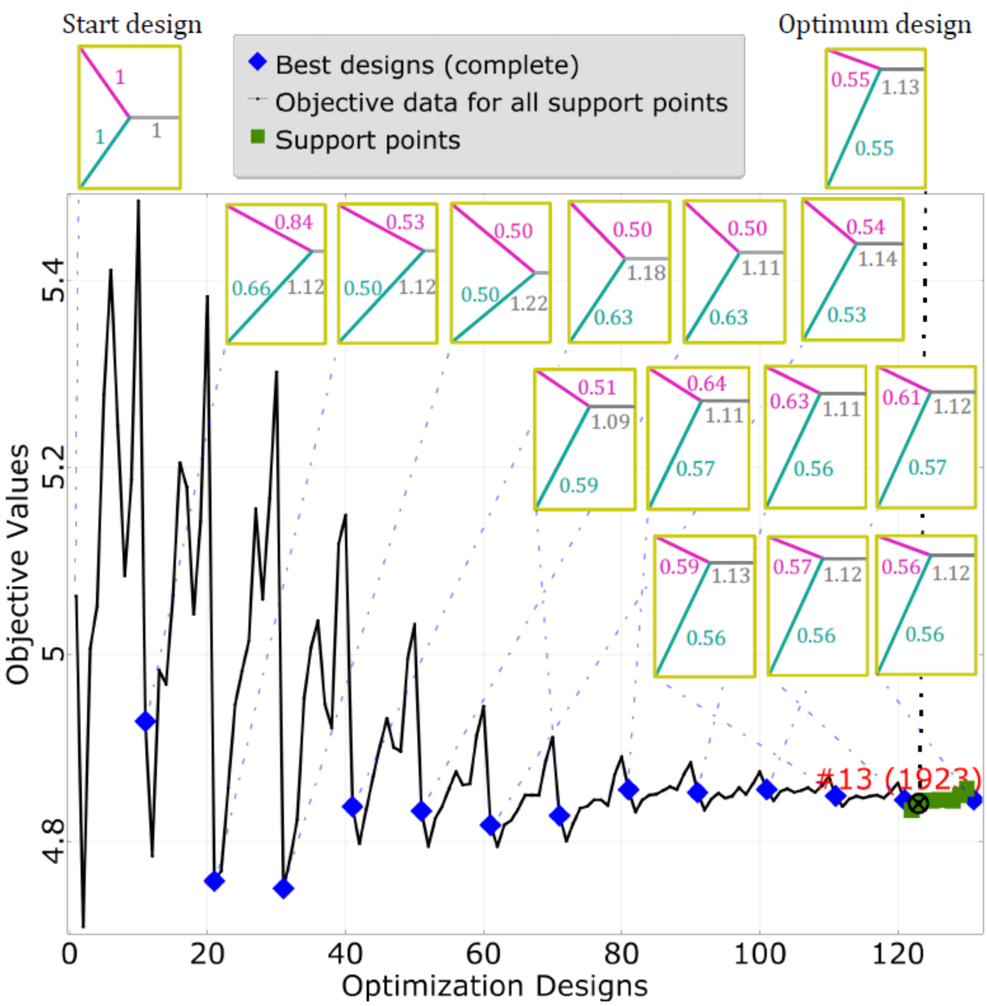

Fig.10: Development of the objective for the illustration example showing the change of interior reinforcements, [10].

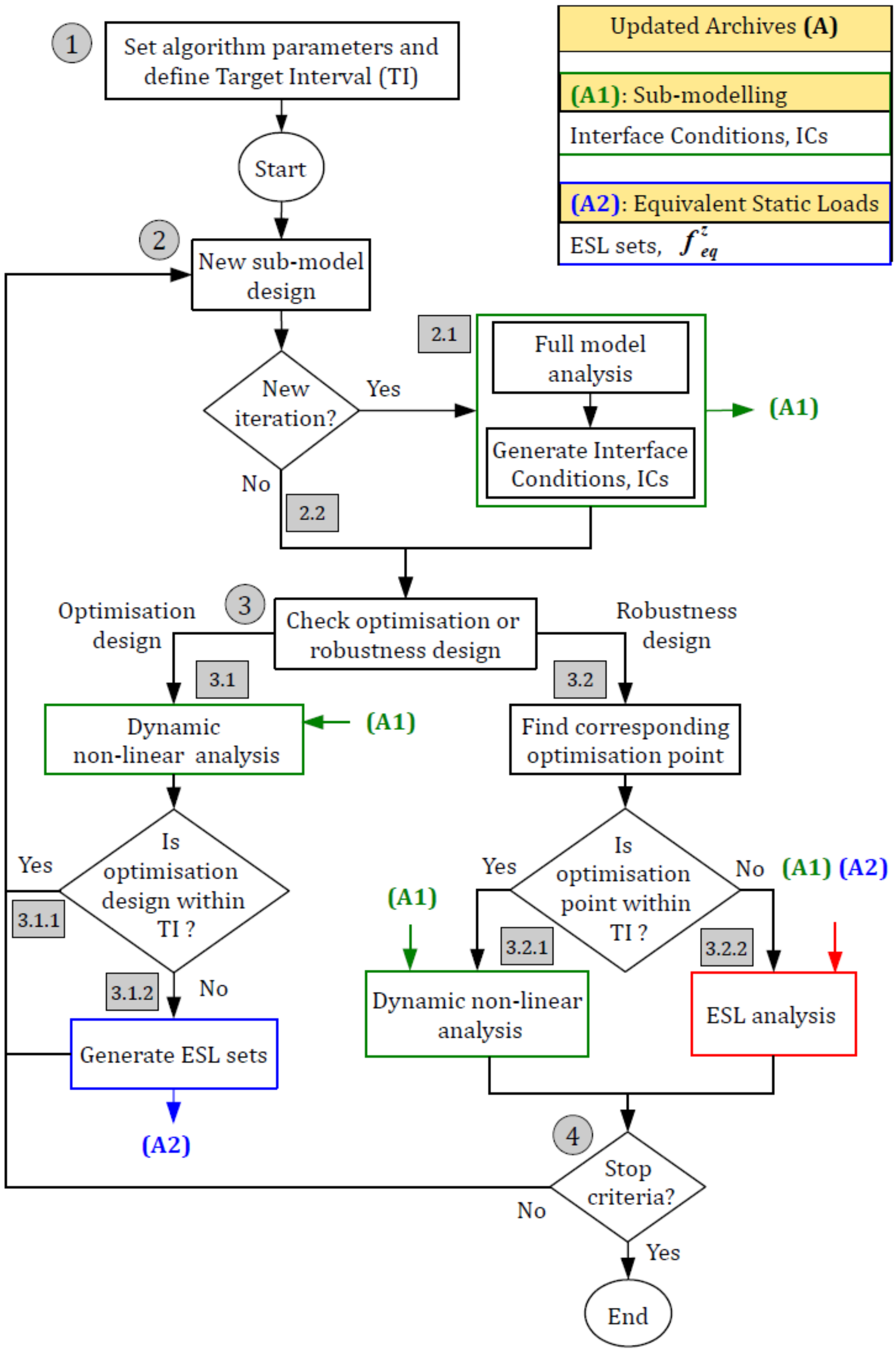

Fig.11: Flow diagram for the combined approach with two physical surrogates (equivalent static loads and sub-structure), [10].

RDO	Analysis point	Analysis type	No. of analysis	CPU time (h/cpu)	Total time
General approach	Optimisation	Full model	131	87	1387
	Robustness	Non-linear	1950	1300	
New approach	Optimisation	Full model	14	9	275
		Sub-model	130	26	
	Robustness	Non-linear	825	165	
		ESLM	1125	75	

Table 2: Computational effort for the illustration example (pole impact on rocker), [10].

5 Summary

In this paper, new methods based on physical surrogates for robustness of structures with respect to crashworthiness are shown. It is proposed here for the early development phases to use a solution space approach combined with lumped mass models as physical surrogate to assure robustness accounting for lack-of-knowledge uncertainties. For later development stages, computational effort can be reduced by employing sub-structures and/or equivalent static loads as physical surrogates.

6 Literature

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