Cosserat Point Elements in LS-DYNA

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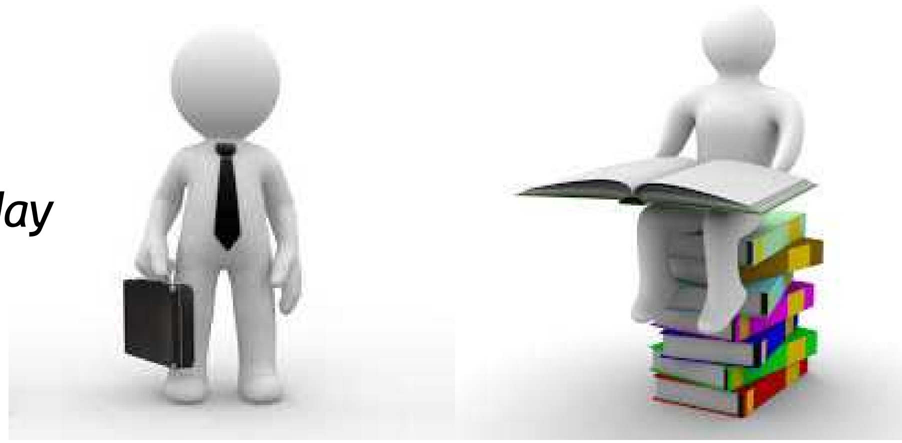
Perspectives of LS-DYNA Development

- The driving force behind LS-DYNA development is two-fold
  - Commercial
    - Customer requests
    - Solve advanced problems *today*
  - Research
    - Resolve fundamental issues
    - Build for the *future*



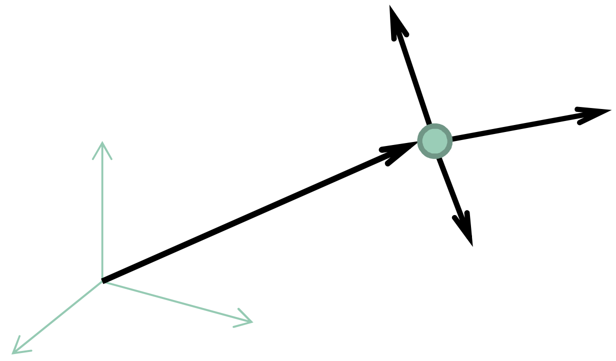
- LSTC is devoting a lot of effort to the latter
  - Major fields of development
    - EFG, SPH, CFD, EM
  - Keep up with current state of research
    - Material Modeling
    - *Element Technology*

Finite Element Technology – History and Challenges

- Research and engineering goes back a long time
  - 1941 – Courant solved the Laplace torsion problem
  - 1956 – Clough et.al. used the idea of elements in determining frequencies of Delta wing aircraft
  - 1960 – named FEM
  - 60's and 70's work lead up to the start of todays commercial softwares
  - And so it goes…
- Why isn't there a *universal* Finite Element?
  - Straightforward derivation from variational formulation of PDE often leads to kinematical restrictions
    - Volumetric and shear locking
  - Various solutions
    - Reduced or selective integration with stabilization (efficient)
    - Higher order elements or assumed strain formulations (expensive)
- Where do the Cosserat Point Elements fit in this context?

The Cosserat Point Element – Basic Idea

- Derived from the balance laws of a *Cosserat Point*



$\mathbf{d}_0$ location of Cosserat Point

$\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3$ homogeneous directors

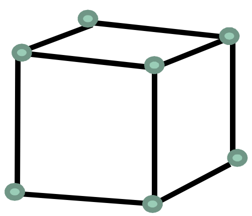
$\mathbf{d}_i \quad i = 4,5,\ldots,n$ inhomogeneous directors

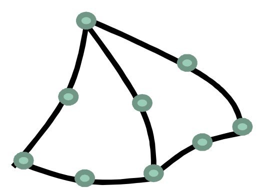
$$d = \mathbf{d}_1 \times \mathbf{d}_2 \cdot \mathbf{d}_3 > 0, \quad \mathbf{F} = \mathbf{d}_i \otimes \mathbf{D}^i, \quad \mathbf{D}_i \otimes \mathbf{D}^i = \mathbf{I}$$

$$\underbrace{\frac{d}{dt}\left(I^{ij}\dot{\mathbf{d}}_j\right) = m\mathbf{b}^i + \mathbf{m}^i - \mathbf{t}^i}_{\text{Linear momentum}} \quad i = 0,\ldots,n, \quad \underbrace{\mathbf{T} = \mathbf{t}^i \otimes \mathbf{d}_i = \mathbf{T}^T}_{\text{Angular momentum}}$$

- The element is defined by a mapping between the nodal coordinates and directors

$$\mathbf{x}_i = A_i^j \mathbf{d}_j \quad i = 0,\ldots,n$$





The Cosserat Point Element – Specifics

- For the mapping **A** chosen it turns out that the following measures homogeneous and inhomogeneous deformations

$$\overline{\mathbf{F}} = \mathbf{F}\left(\mathbf{I} + \boldsymbol{\beta}_i \otimes \mathbf{V}^i(\mathbf{D}_j)\right)$$ Element averaged deformation gradient

$$\boldsymbol{\beta}_i = \mathbf{F}^{-1}\mathbf{d}_{i+3} - \mathbf{D}_{i+3} \quad i = 1,\ldots,n-3$$

- The constitutive law is hyperelastic defined by

$$W = W(\underbrace{\overline{\mathbf{F}}^T\overline{\mathbf{F}}}_{\overline{\mathbf{C}}}, \boldsymbol{\beta}_i) \Rightarrow$$

$$\Rightarrow$$

$$\mathbf{T} = 2\overline{\mathbf{F}}\frac{\partial W}{\partial \overline{\mathbf{C}}}\overline{\mathbf{F}}^T$$ Element averaged Cauchy stress

$$\mathbf{t}^{i+3} = \mathbf{F}^{-T}\frac{\partial W}{\partial \boldsymbol{\beta}_i}$$ Hourglass resistance

The Cosserat Point Element – Final Touch

- The elastic energy density W is split additively

$$W = W_1(\overline{\mathbf{C}}) + W_2(\boldsymbol{\beta}_i)$$

W_1 is the $3D$ elastic strain energy

$$W_2 = \frac{1}{2}\boldsymbol{\beta}_i^T \mathbf{B}^{ij} \boldsymbol{\beta}_j \text{ is the hourglass energy}$$

- The coefficients in the hourglass energy are determined to match small deformation solutions to elementary cases (bending and torsion)
- They depend on the reference configuration as well as elastic moduli

$$B_{11}^{11} = B_{11}^{11}(\mathbf{D}_i)$$

$$B_{21}^{11} = B_{21}^{11}(\mathbf{D}_i)$$

$$B_{31}^{11} = B_{31}^{11}(\mathbf{D}_i)$$

$$\vdots$$

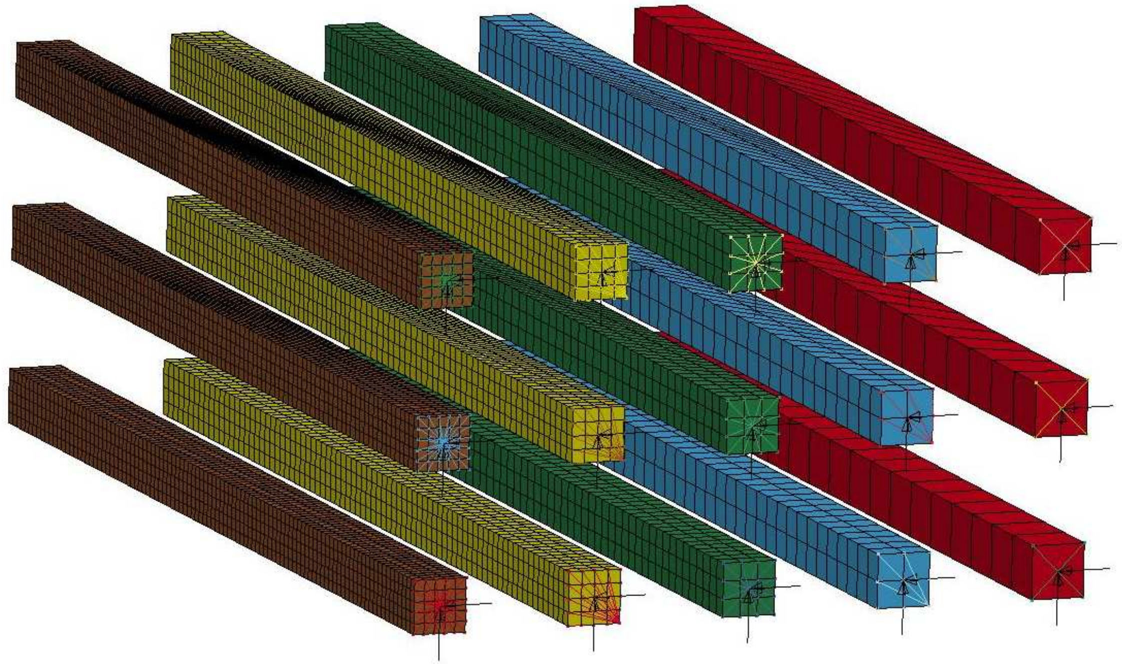
The Cosserat Point Element – Comments

- The element is not derived from a continuum approach but is to be seen as a structure
- The kinematic approximation is not valid point wise, instead averaged quantities are used
- Nonlinear patch tests are satisfied exactly and analytically
- The elements are invariant to rigid body motion
- The element is made generic in extending W_1 to general hypo formulation by exploiting

$$\overline{\mathbf{L}} = \dot{\overline{\mathbf{F}}}\,\overline{\mathbf{F}}^{-1}$$

  while the hourglass part remains the same
- The tetrahedron jacobian is modified to account for exact volume and thereby stabilize the element

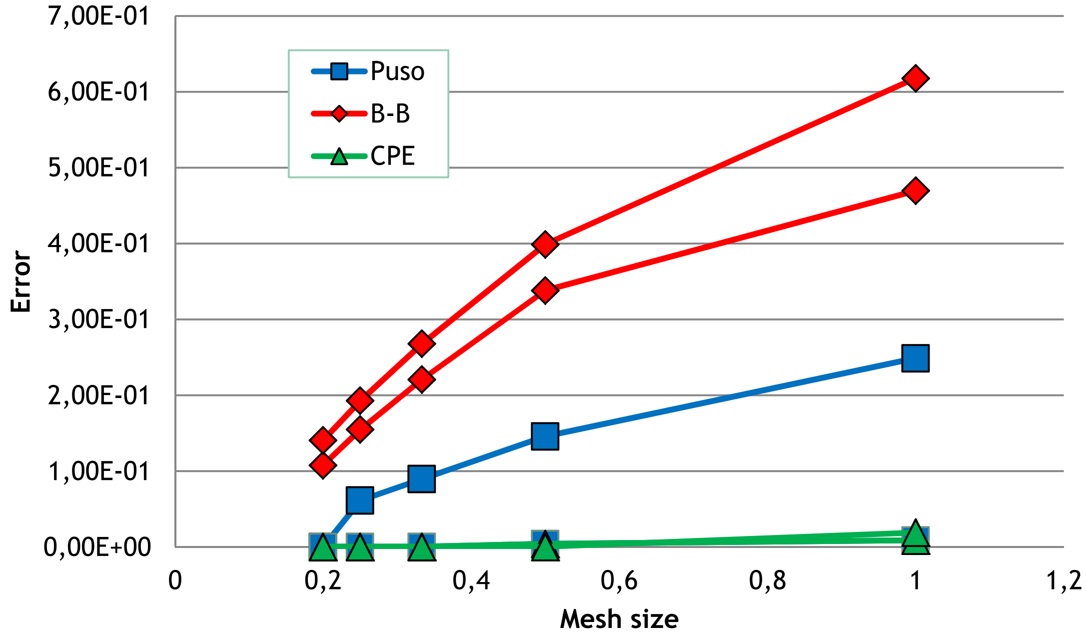
Mesh Sensitivity of Hexahedron



- Tip loaded cantilever beam with small displacements
  - 5 mesh size levels (H=10, 5, 3.33, 2.5, 2 mm)
  - 3 distortion levels (a=-20, 0, 20 mm)
  - 2 load cases (horizontal (H) and vertical (V))
- Analytical tip displacement 0.21310 mm

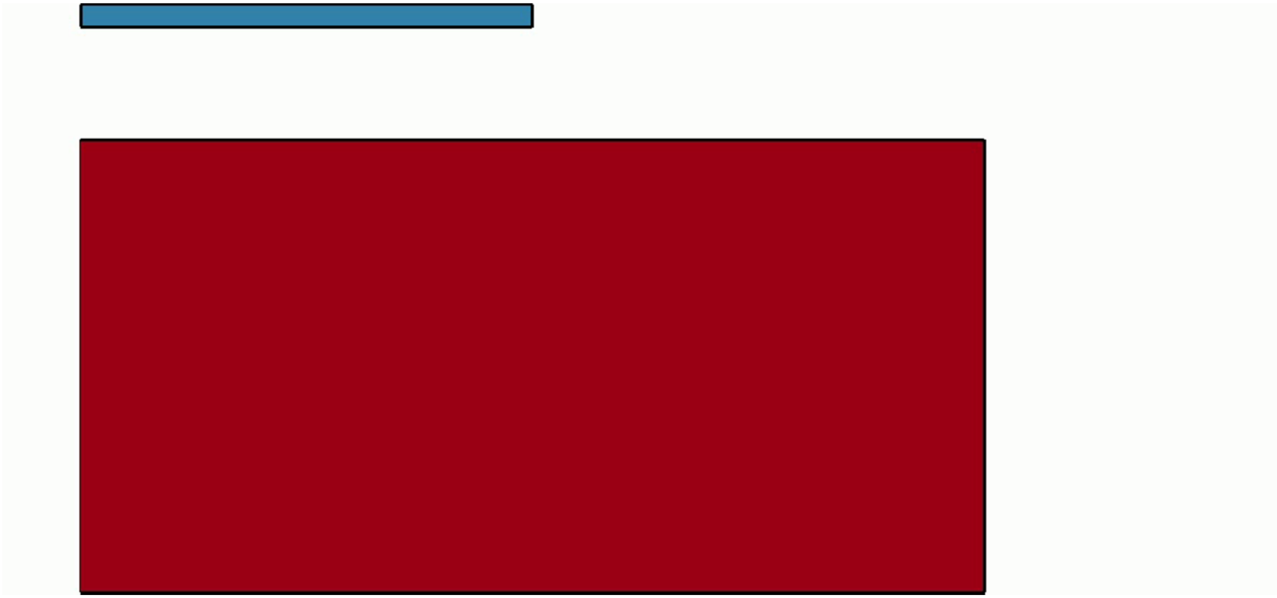
Mesh Convergence Results

CPE	B-B	Puso
1.7%	61.8%	24.8%
0.8%	46.8%	14.7%
0.6%	40.0%	14.5%
0.3%	39.8%	9.2%
0.2%	33.9%	8.5%
0.2%	27.0%	6.2%
0.1%	24.6%	5.3%
0.1%	22.3%	3.6%
0.1%	19.0%	0.9%
0.1%	15.4%	0.3%



- Cosserat Point Element is much less mesh sensitive than Belytschko-Bindeman and Puso elements

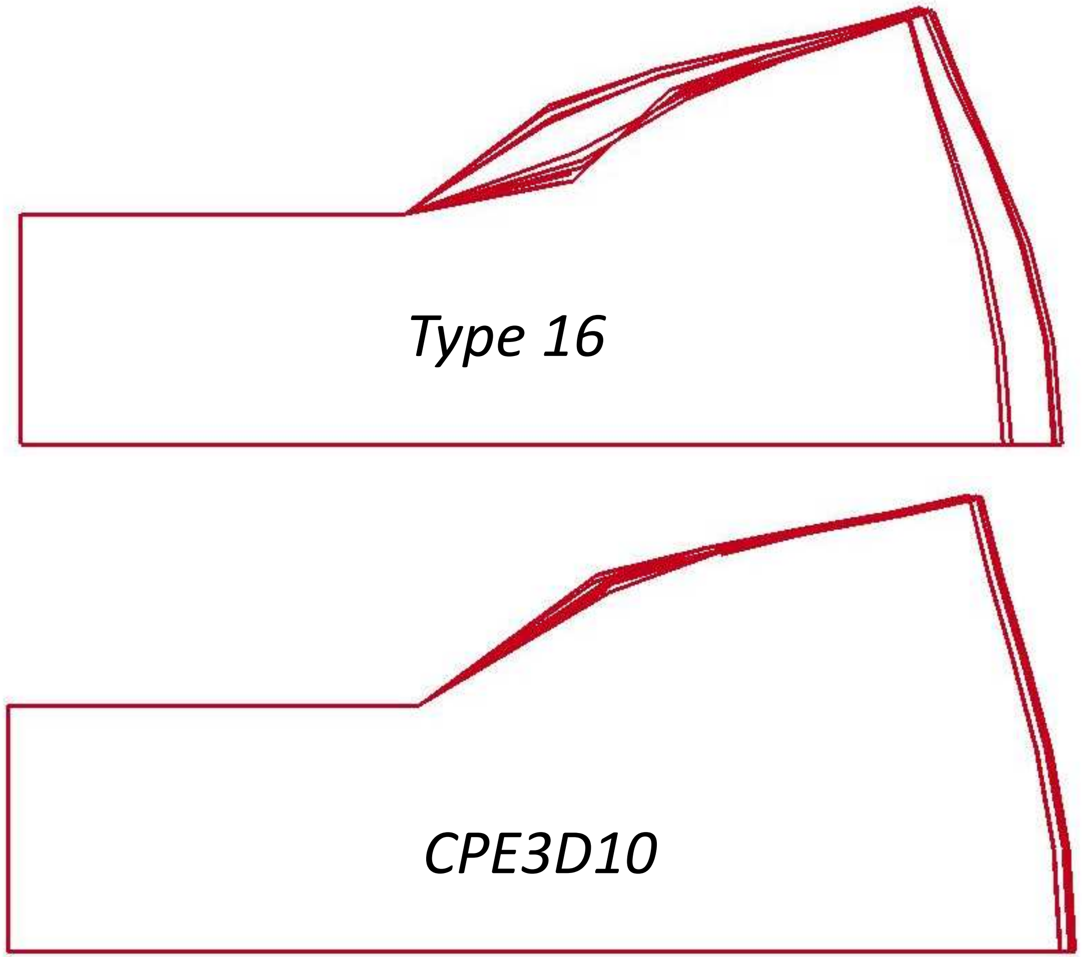
Mesh Sensitivity and Robustness of Tetrahedron



- Plane strain
- Implicit with extremely tight convergence criterion
- Hyperelastic rubber (PR=0.4997)
- 5 different mesh orientations
- CPE3D10 vs. Type 16 (NIP=4)

- Three basic checks
  - Sensitivity of results with respect to mesh orientation
  - How far can the block be compressed
  - How many iterations and reformations are needed

Check #1 – Mesh Sensitivity



- Overlays of the different meshes for 50% compression

Check #2 and #3 – Robustness and Convergence

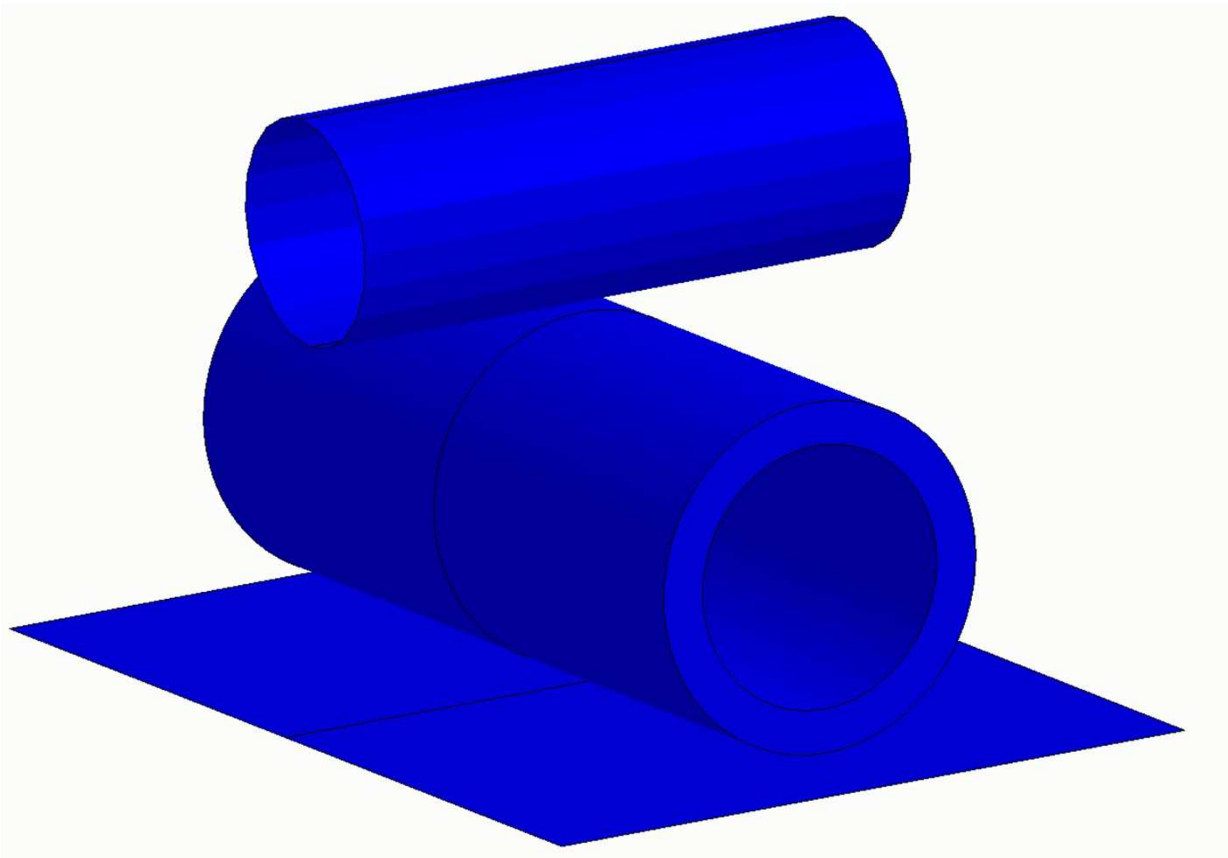
- CPE3D10

Max % comp.	Vol % error	Iter/Ref
56.5	0.5	900/57
61.5	0.6	883/55
51.5	0.4	883/56
40	0.3	858/50
51	0.5	882/56

- Type 16

Max % comp	Vol % error	Iter/Ref
29	0.5	1562/110
35.5	1.6	983/61
32	0.5	1237/84
32.5	0.8	1031/66
35	0.6	1162/77

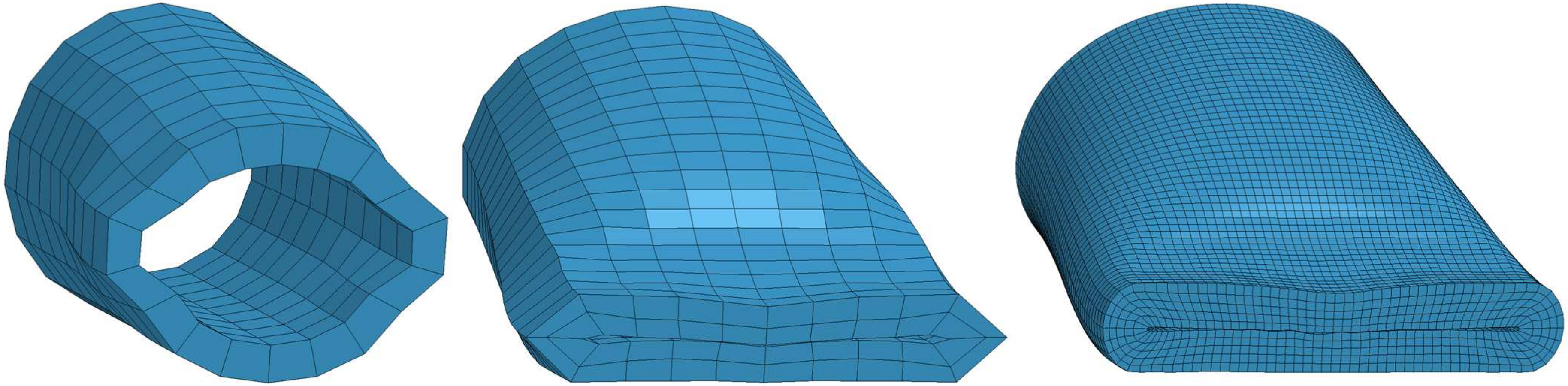
Elastoplastic Response for Hexahedron and Tetrahedron



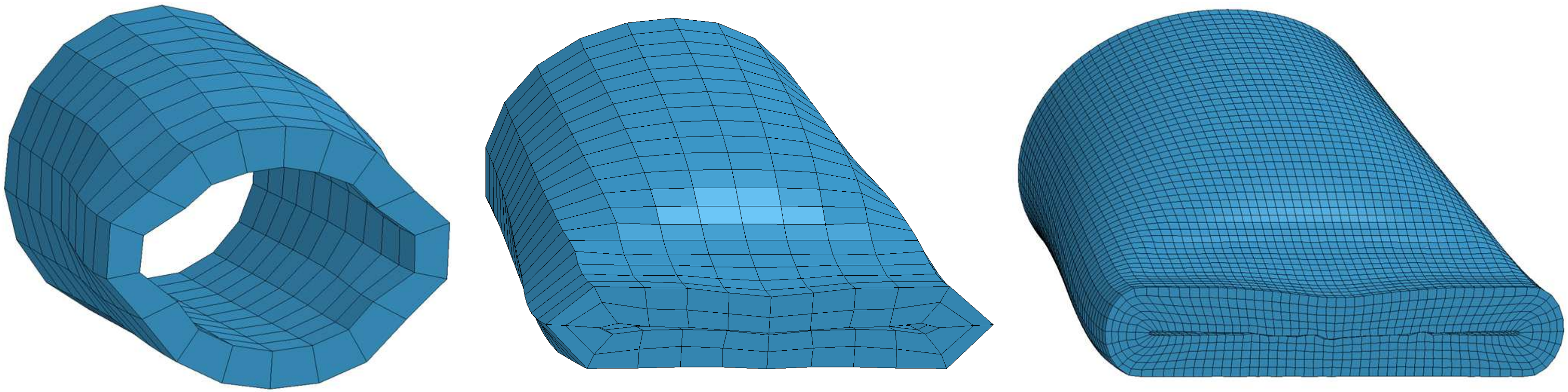
- Quasistatic indentation of cylinder
- Run with tetrahedra and hexahedra
- Monitor contact force and examine final configuration

Hexahedron Results

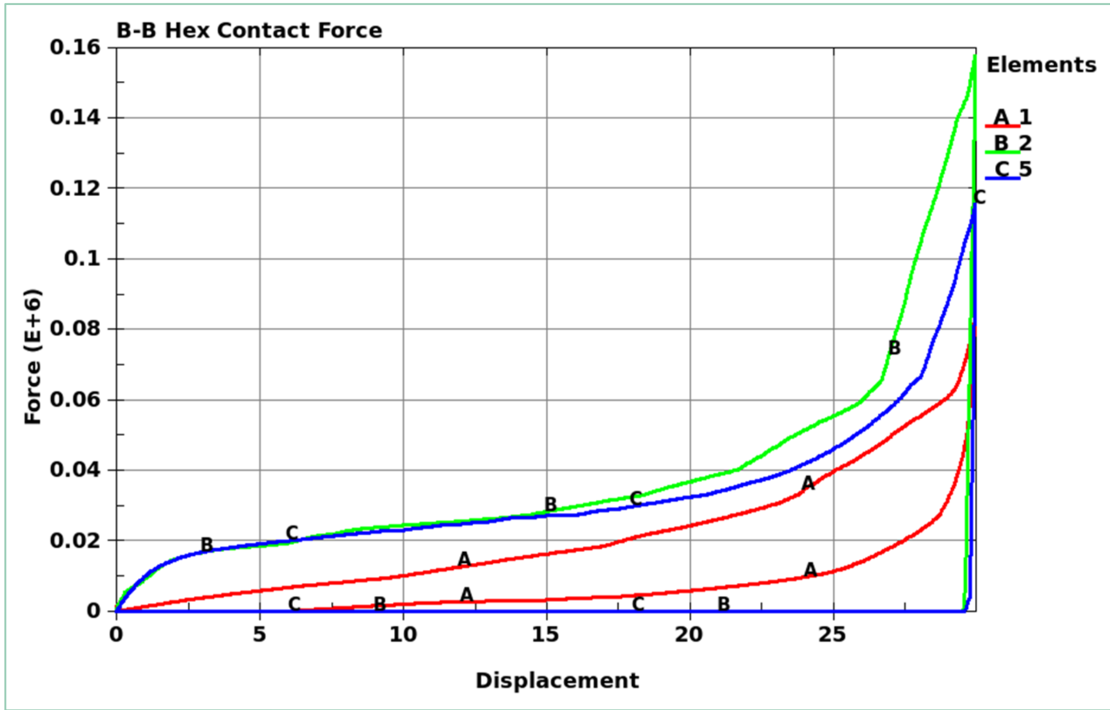
- Belytschko-Bindeman Hourglass



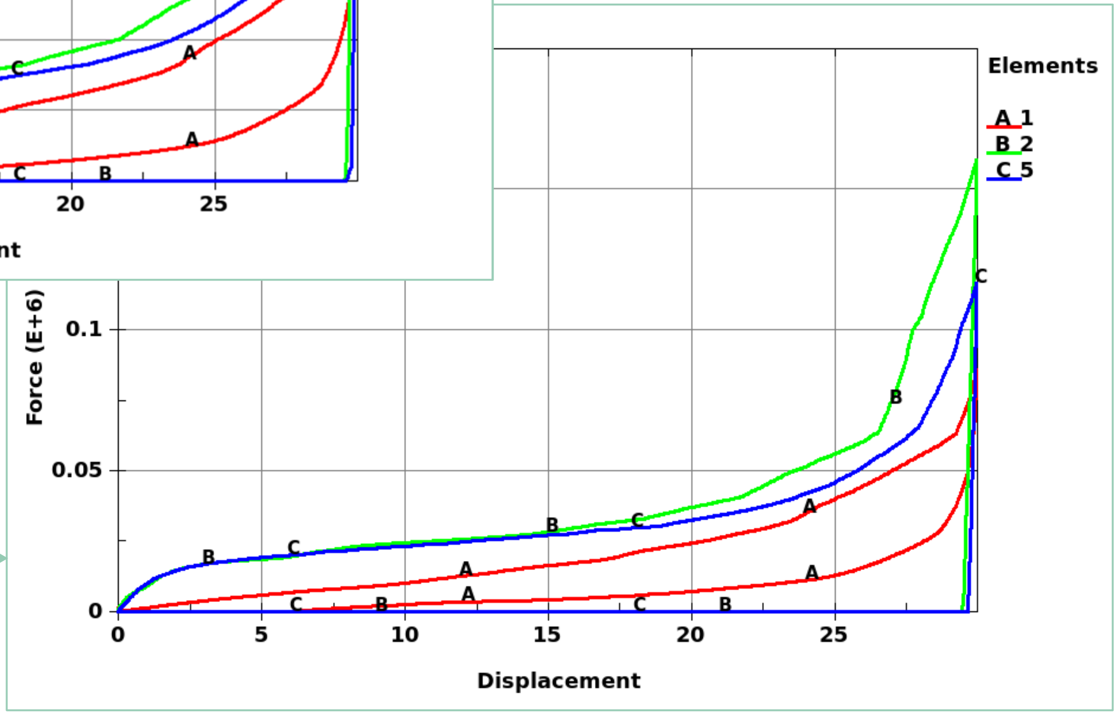
- Cosserat Point Element



Hexahedron Contact Forces



Results very similar, hysteresis for coarse mesh due to excessive hourglass energy

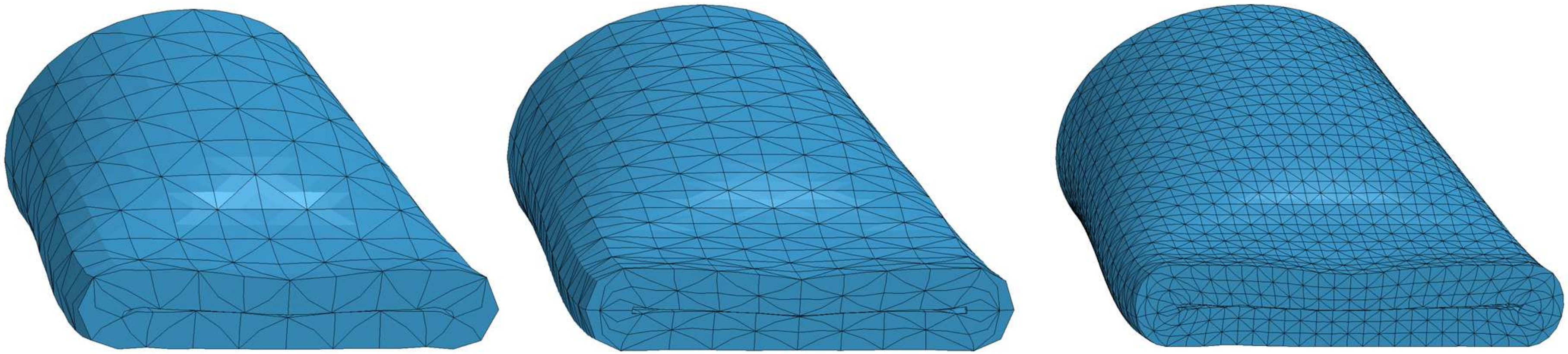


Belytschko-Bindeman

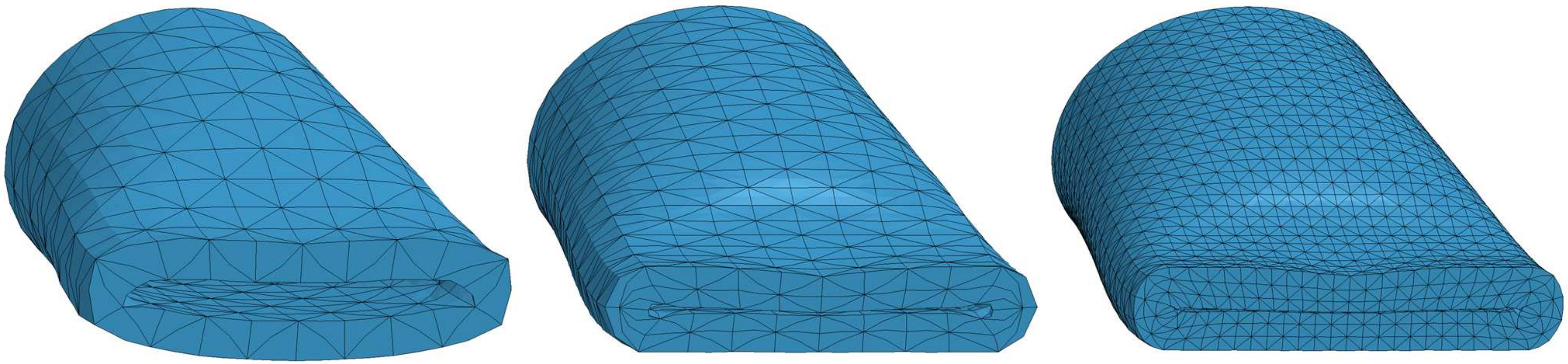
Cosserat Point

Tetrahedron Results

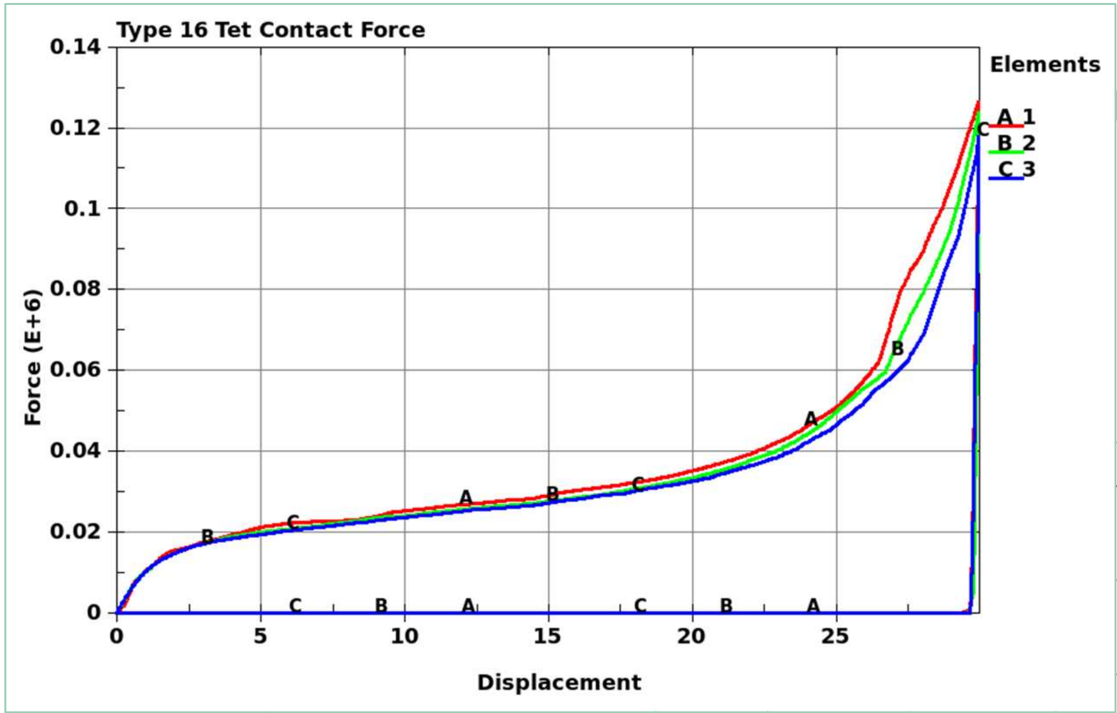
- Fully Integrated Tetrahedron (type 16)



- Cosserat Point Element



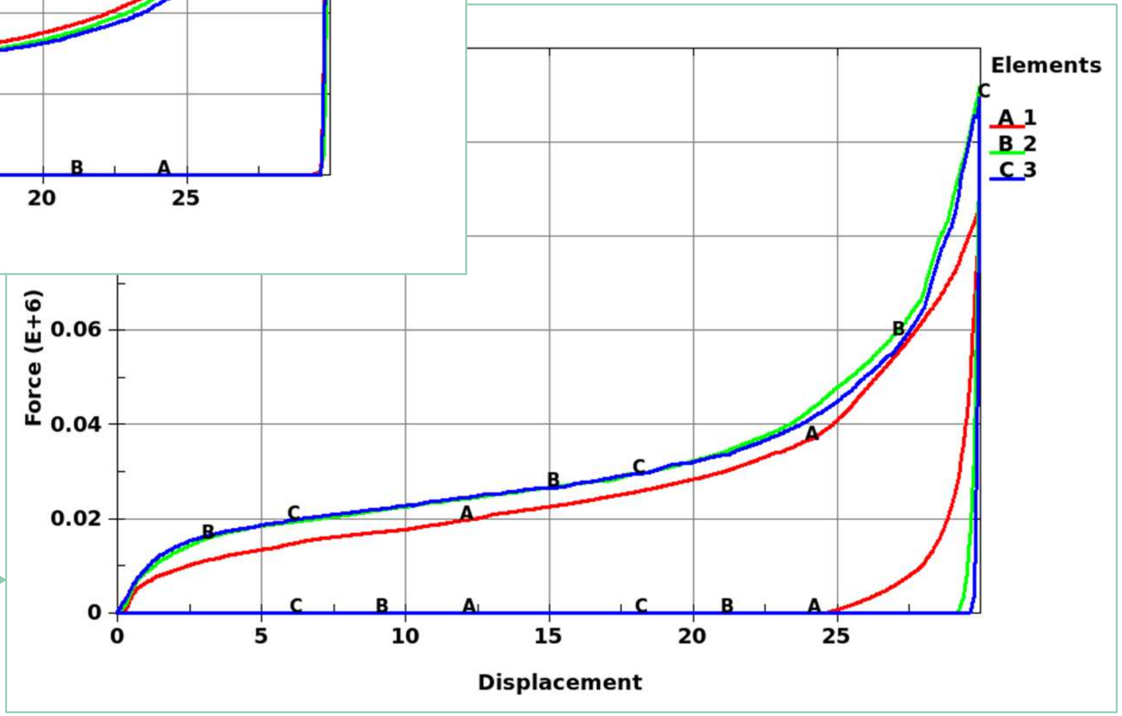
Tetrahedron Contact Forces



Stronger mesh dependence for CPE tetrahedron in elastoplasticity

Full Integration

Cosserat Point

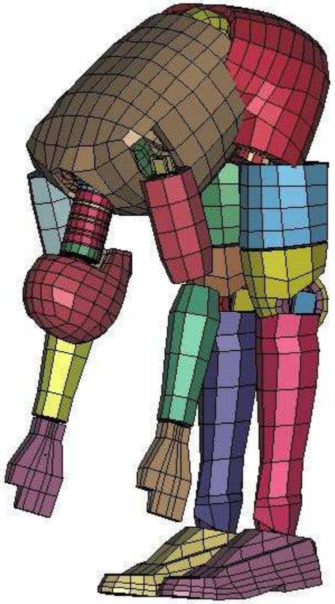


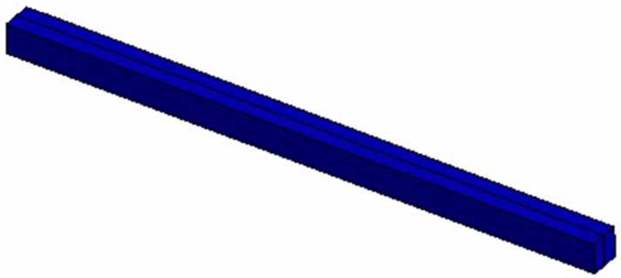
Summary

- The Cosserat Point Element can be seen as
  - A structure where kinematical quantities are averaged or…
  - Reduced ”integrated” element with stabilization, even for the high order tetrahedron
- Theory is derived in hyperelastic context
  - Use 1.0 as hourglass coefficient for elastic materials
  - Reduce hourglass coefficient for elastic-plastic materials
- In LS-DYNA it is implemented as hourglass type 10 and applies to
  - Hexahedron element type 1
  - Tetrahedron element type 16
- It provides high accuracy and insignificant mesh sensitivity within the derived theory, whereas for inelastic materials the results are affected by the stabilization procedure

(Psst! Use Implicit)

Thank you!





Folding beam with follower force taken from

Jabareen, M., and Rubin, M.B., A Generalized Cosserat Point Element (CPE) for Isotropic Nonlinear Elastic Materials including Irregular 3-D Brick and Thin Structures, J. Mech. Mat. And Struct., Vol 3-8, 1465-1498 (2008).

Jabareen, M., Hanukah, E. and Rubin, M.B., A Ten Node Tetrahedral Cosserat Point Element (CPE) for Nonlinear Isotropic Elastic Materials, J. Comput. Mech. 52, 257-285 (2013).